TEACHERS FORUM®

QUESTION BANK

(solved)

Class IX

MATHEMATICS

SUBJECT EXPERTS

CONTENTS

1 NUMBER SYSTEM

NCERT SOLUTIONS

EXERCISE 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$?

Ans. Yes, zero is a rational number.

Example : $\frac{0}{1}, \frac{0}{-2}$

Which is in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$

2. Find six rational numbers betwen 3 and 4.

Ans. $3 = 3 \times \frac{10}{10} = \frac{30}{10}$ and $4 = 4 \times \frac{10}{10} = \frac{40}{10}$

$\therefore$ Six rational numbers are $\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}, \frac{36}{10}$

3. Find five rational numbers between 3/5 and 4/5.

Ans. $\frac{3}{5} = \frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$

$\frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

$\therefore$ Five rational numbers are $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Ans. (i) True, because the set of natural numbers is represented as N = {1, 2, 3……} and the set of whole numbers is W = {0, 1, 2, 3 ………}. We see that every natural number is present in the set of whole numbers.

(ii) False. Negative integers are not present in the set of whole numbers.

(iii) False. For example, $\frac{1}{5}$ is a rational number, which is not a whole number.

EXERCISE 1.2

1. State whether the following statements are true or false. Justify your Answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where 'm' is a natural number.

(iii) Every real number is an irrational number.

Ans. (i) True, because the set of real numbers consists of rational numbers and irrational numbers.

(ii) False, for example $\sqrt{\frac{2}{3}}$ is a real number on the number line but $\frac{2}{3}$ is not a natural number.

(iii) False, for example $\frac{1}{2}$ is a rational number and hence it is real. But it is not an irrational number.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Ans. The square roots of all positive integers are not irrationals.

Example: $\sqrt{25}$ = 5 and 5 in a rational number ($\because 5 = \frac{5}{1}$)

3. Show how $\sqrt{5}$ can be represented on the number line.

Ans. Take OA = 2unit

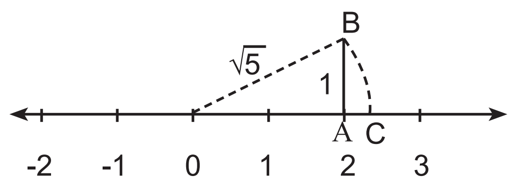
Draw BA $\perp$ OA, such that BA = 1 unit

Join OB.

Now OB = $\sqrt{2^2 + 1^2} = \sqrt{5}$

Taking OB as radius, draw an arc to meet at 'C'

i.e., C represents $\sqrt{5}$.



EXERCISE 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Ans. (i) $\frac{36}{100}$ = 0.36. Terminating decimal

(ii) $\frac{1}{11}$

```
       0.0909
11 | 1.00
   |   99
   | -----
   |   100
   |    99
     ----
       1
```

The remainder 1 keeps repeating. $\frac{1}{11}$ = 0.0909 and can be written as $\frac{1}{11} = 0.\overline{09}$

Non-terminating recurring decimal.

(iii) $4\frac{1}{8} = \frac{33}{8}$

```
     4.125
8 | 33.0
  | 32
  | -----
  | 10
  |  8
  ----
     20
     16
    ----
     40
     40
    ----
      0
```

$4\frac{1}{8} = 4.125$

Terminating decimal (∵ The remainder is zero)

(iv) $\frac{3}{13} = 0.23076923$

```
      0.23076923
13 |          30
   |          26
   |       -----
   |          40
   |          39
             ----
             100
              91
             ----
              90
              78
             ----
             120
             117
            -----
              30
              26
             ----
              40
              39
             ----
               1
```

∵ We find the block of numbers 230769 keep repeating.

This is non-terminating recurring decimal and is written as:

$\frac{3}{13} = \overline{0.230769}$

(v) $\frac{20}{11} = 0.1818$

```
      0.1818
11 | 20
   | 11
   | ------
   | 90
   | 88
    ----
      20
      11
    ----
      90
      88
    ----
       2
```

Here we find the block of numbers 18 keep repeating. Hence this is a non-terminating recurring decimal and is written as : $\frac{2}{11} = 0.\overline{18}$

(vi) $\frac{329}{400} = \frac{329}{4 \times 100}$

$$\begin{array}{r|l} & 82.25 \\ 4 & 329 \\ & 32 \\ & \text{-----} \\ & 09 \\ & 08 \\ & \text{----} \\ & 10 \\ & 8 \\ & \text{----} \\ & 20 \\ & 20 \\ & \text{----} \\ & 0 \end{array}$$

$\frac{82.25}{100} = 0.8225$

Terminating decimal

($\because$ The remainder is zero)

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{3}{7}$,$\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ are, without actually doing the long division? If so, how?

Ans. $\frac{1}{7} = 0.\overline{142857}$

$$\begin{array}{r|l} & 0.142857 \\ 7 & 10 \\ & 7 \\ & \text{-----} \\ & 30 \\ & 28 \\ & \text{----} \\ & 20 \\ & 14 \\ & \text{----} \\ & 60 \\ & 56 \\ & \text{----} \\ & 40 \\ & 35 \\ & \text{----} \\ & 50 \\ & 49 \\ & \text{----} \\ & 1 \end{array}$$

This is a non-terminating recurring decimal.

We can use this to find the decimal expansion of

$\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$

To write the decimal expansion for

(i) $\frac{2}{7}$: We observe that we get 2 as remainder after the second step in the above division. Hence, we start writing the quotient after the second decimal place and we get $\frac{2}{7} = 0.\overline{285714}$

(ii) $\frac{3}{7}$: 3 is the remainder after the first step. Hence $\frac{3}{7} = 0.\overline{428571}$

(iiii) $\frac{4}{7}$: 4 is the remainder at the 4^{th} step. Hence $\frac{4}{7} = 0.\overline{571428}$

(iv) $\frac{5}{7}$: 5 is the remainder at the 5th step. Hence $\frac{5}{7} = 0.\overline{714285}$

(v) $\frac{6}{7}$: 6 is the remainder after the 3rd step. Hence $\frac{6}{7} = 0.\overline{857142}$

3. Express the following in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Ans. (i)

$$\text{Let } x = 0.666\ldots \quad \rightarrow(1)$$

$$10x = 6.666\ldots \quad \rightarrow(2)$$

(2) - (1) $\Rightarrow$

$$9x = 6$$

$$x = \frac{6}{9} = \frac{2}{3}$$

$$\text{i.e., } 0.\overline{6} = \frac{2}{3}$$

(ii)

$$\text{Let } x = 0.4777\ldots$$

$$100x = 47.777\ldots \quad \rightarrow(1)$$

$$10x = 4.777\ldots \quad \rightarrow(2)$$

(1) - (2) $\Rightarrow$

$$90x = 43$$

$$x = \frac{43}{90}$$

$$\text{i.e., } 0.4\overline{7} = \frac{43}{90}$$

(iii)

$$\text{Let } x = 0.001001\ldots \quad \rightarrow(1)$$

$$1000x = 1.001001\ldots \quad \rightarrow(2)$$

(1) - (2) $\Rightarrow$

$$999x = 1$$

$$x = \frac{1}{999}$$

$$\text{i.e., } 0.\overline{001} = \frac{1}{999}$$

4. Express 0.99999… in the form of $\frac{p}{q}$. Are you surprised with your Answer?

With your teacher and classmates discuss why the answer makes sense?

Ans. Let $x = 0.99999\ldots \quad \rightarrow(1)$

Since one digit is repeated.

We should multiply both the sides of (1) by 10

$$10x = 9.9999$$

$$10x = 9 + 0.9999$$

$$10x = 9 + x$$

$10x - x = 9$

$9x = 9 \Rightarrow x = 1$

Hence 0.99999 = 1

5. What can be the maximum number of digits be in the repeating block of digits in decimal expansion of $\frac{1}{17}$? Perform the division to check your Answer.

Ans. Let us perform the division 1 ÷ 17

```
       0.0588235294117647
   17 | 100
      |  85
      | ----------------------------
      |   150
      |   136
      | ----------------
           140
           136
          ---------------
            40
            34
          ---------------
            60
            51
          ---------------
            90
            85
          ---------------
            50
            34
          ---------------
           160
           153
          ---------------
            70
            68
          --------------
            20
            17
          --------------
            30
            17
          --------------
           130
           119
          --------------
           110
           102
          --------------
            80
            68
          --------------
           120
           119
           ----------
             1
```

$\therefore \frac{1}{17} = 0.\overline{0588235294117647}$

There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$

6. Look at the several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$) where p and q are integers with no common factors other than 1 and having terminating decimal

representation (expansions). Can you guess what property q must satisfy?

Ans. We shall look at some examples of rational numbers in the form of $\frac{p}{q}$ (q ≠ 0) where decimal representations are terminating.

$\frac{2}{5} = 0.4$ $\frac{7}{100} = 0.07$ $\frac{27}{16} = 1.6875$ $\frac{11}{50} = 0.22$

We observed that the denominators of above rational numbers are in the form of 2^m x 5^n, where, a and b are whole numbers.

Hence if q is in the form 2^m x 5^n then $\frac{p}{q}$ is a terminating decimal.

7. Write three numbers whose decimal expansions are non terminating and nonrecurring.

Ans. (i) 0.212212221… (ii) 0.03003000300003… (iii) 0.825882588825…

8. Find three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$

Ans. Let us find the decimal expansion of $\frac{5}{7}$ and $\frac{9}{11}$.

```
      0.714285
 7 |  50
      49
      -----
      10
       7
      ----
       30
       28
      ----
       20
       14
      ----
        60
        56
      ----
        40
        35
      ----
         5
```

```
       0.81
 11 |  90
       88
      -----
        20
        11
       ----
         9
```

We can write 3 irrational numbers between them as follows:

(i) 0.731733173331… (ii) 0.750975009750009…

(iii) 0.808008000…

9. Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796 (iv) 7.478478 ... (v) 1.101001000100001….

Ans. (i) $\sqrt{23} = \frac{\sqrt{23}}{1} = \frac{p}{q}$, but p is not an integer.

Hence $\sqrt{23}$ is an irrational number

(ii) $\sqrt{225} = \frac{15}{1} = \frac{p}{q}$, where p and q are integers. q ≠ 0.

Hence, $\sqrt{225}$ is a rational number.

(iii) 0.3796

0.3796 is a rational number. Because, it is a terminating decimal number.

(iv) 7.478478.... = $7.\overline{478}$

It is a rational number. Because, it is a non-terminating recurring decimal.

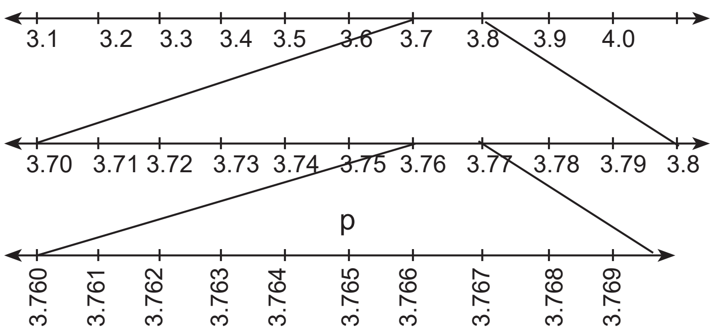
(v) 1.101001000100001…….

It is an irrational number. Because, it is a non-terminating and non-recurring decimal.

EXERCISE 1.4

1. Visualise 3.765 on the number line using successive magnification.

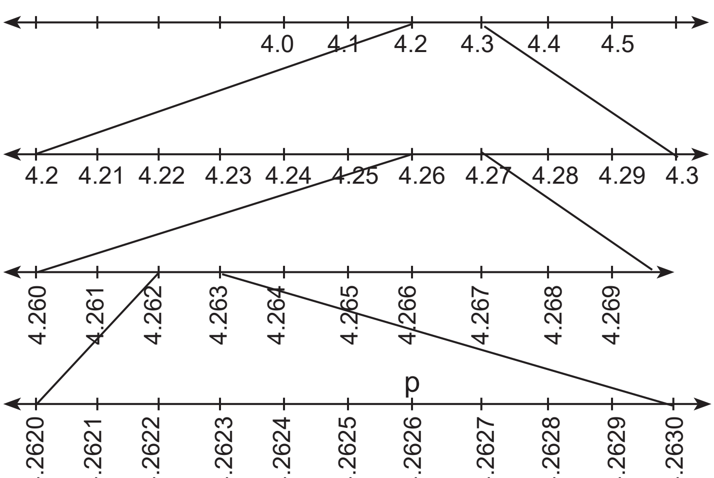
Ans.



The point 'P' represents 3.765.

2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Ans. $4.\overline{26}$ = 4.2626.....



The point 'P' represents $4.\overline{26}$.

EXERCISE 1.5

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$ (ii) $(3 + \sqrt{23}) - \sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Ans. (i) The sum or difference of a rational number and an irrational number is always irrational.

Here 2 is a rational number and $\sqrt{5}$ is an irrational number. Hence $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23} = 3$

$3 = \frac{1}{3}$, which is in the form of $\frac{p}{q}$. Hence is $(3 + \sqrt{23}) - \sqrt{23}$ a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$ which is in the form of $\frac{p}{q}$. Hence $\frac{2\sqrt{7}}{7\sqrt{7}}$ a rational number.

(iv) $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) $\pi = 3.1415$ ....

π is an irrational number whose value is non-terminating and non-recurring.

2 is a rational number.

Product of a non-zero rational number and irrational number is an irrational number.

Hence 2π is irrational.

2. Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$ (ii) $(3 + \sqrt{3})(3 - \sqrt{3})$ (iii) $(\sqrt{5} + \sqrt{2})^2$ (iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Ans. (i) $(3 + \sqrt{3})(2 + \sqrt{2}) = (6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6})$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3}) = 3^2 - (\sqrt{3})^2 = 9 - 3 = 6$

(iii) $(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + 2 \text{ x } \sqrt{5} \text{ x } \sqrt{2} + (\sqrt{2})^2 = 5 + 2\sqrt{10} + 2 = 7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = \sqrt{5}^2 - (\sqrt{2})^2 = 5 - 2 = 3$

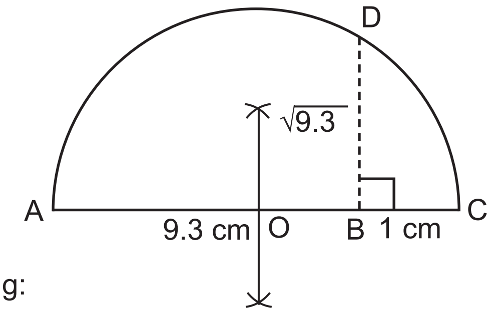
3. Recall, π is defined as the ratio of circumference (say c) to its diameter (say d). That is $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Ans. Writing π as $\frac{22}{7}$ is only an approximate value and so we can't conclude that it is in the form of a rational. In fact, the value of π is non-terminating, non-recurring decimal as $\pi = 3.14159$.

4. Represent $\sqrt{9.3}$ on the number line.

Ans. Draw AC = 10.3 cm (9.3 + 1). Now find the centre of AC and draw a semicircle with

AC as diameter. Now draw BD⊥AC to meet the semicircle at point D. Then BD = $\sqrt{9.3}$ cm.



5. Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Ans. (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$

EXERCISE 1.6

1. Find : (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Ans. (i) $64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = 8$ (ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2$ (iii) $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5$

2. Find : (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Ans. (i) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^3 = 27$ (ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^2 = 4$

(iii) $16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = 8$ (iv) $125^{\frac{-1}{3}} = (5^3)^{\frac{-1}{3}} = 5^{-1} = \frac{1}{5}$

3. Simplify : (i) $2^{\frac{2}{3}} . 2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}} . 8^{\frac{1}{2}}$

Ans. (i) $2^{\frac{2}{3}} . 2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{13}{15}}$ (ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{21}} = 3^{-21}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{1}{4}}$ (iv) $7^{\frac{1}{2}} . 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$

ADDITIONAL QUESTIONS AND ANSWERS

EXERCISE 1.1

Topics : Irrational Numbers, Real Numbers and their Decimal Expansions
Representing Real Numbers on the Number Line.

2 MARKS

1. Express -0.00875 in the form of $\frac{p}{q}$, where p and q are integers and q ≠ 0. **(2016)**

Ans. $-0.00875 = \frac{-875}{100000} = \frac{-35}{4000} = \frac{-7}{800}$

2. Is zero (0) a rational number ? Justify your answer. **(2015)**

Ans. Yes, zero is a rational number.

Zero can be expressed as, $\frac{0}{5}, \frac{0}{26}, \frac{0}{100}$ etc, which are in the form of $\frac{p}{q}$, where p and q are integers and q ≠ 0.

3. Find two rational numbers between 0.121221222122221….and 0.141441444…in the $\frac{p}{q}$ form, where p and q are integers and q ≠ 0. **(2016)**

Ans. Two rational numbers between 0.12122 ..... and 0.14144 are 0.13 and 0.14

i.e., $\frac{13}{100}$ and $\frac{14}{100}$

4. Write $\frac{3}{13}$ in decimal form and state what kind of decimal expansion does it have? **(2016, 2014)**

Ans. $\frac{3}{13} = 0.\overline{230769}$

$$\begin{array}{r|l} & 0.2307692 \\ 13 & 30 \\ & 26 \\ \hline & \;\;40 \\ & \;\;39 \\ \hline & \;\;\;100 \\ & \;\;\;\;91 \\ \hline & \;\;\;\;\;90 \\ & \;\;\;\;\;78 \\ \hline & \;\;\;\;120 \\ & \;\;\;\;117 \\ \hline & \;\;\;\;\;\;30 \\ & \;\;\;\;\;\;26 \\ \hline & \;\;\;\;\;\;\;4 \end{array}$$

It has a non-terminating recurring decimal expansions

5. Insert three rational numbers betwee $\frac{3}{5}$ and $\frac{5}{7}$. **(2014)**

Ans. $\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$

$$\frac{5}{7} \times \frac{5}{5} = \frac{25}{35}$$

$\therefore$ Three rational numbers are $\frac{22}{35}, \frac{23}{35}, \frac{24}{35}$

6. Express $0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **(2011, 2014)**

Ans.

$$\text{Let } x = 0.4777 \ldots$$

$$100x = 47.777 \ldots \quad \rightarrow(1)$$

$$10x = 4.777 \ldots \quad \rightarrow(2)$$

(1) - (2) $\Rightarrow$ $90x = 43$

$$x = \frac{43}{90}$$

$$\text{i.e., } 0.4\overline{7} = \frac{43}{90}$$

7. Express $18.\overline{48}$ in the form of $\frac{p}{q}$, where p and q are integers. **(2014)**

Ans.

$$\text{Let } x = 18.4848 \ldots..$$

$$100x = 1848.4848 \ldots \quad \rightarrow(1)$$

$$x = 18.4848 \ldots \quad \rightarrow(2)$$

(1) - (2) $\Rightarrow$ $99x = 1830$

$$x = \frac{1830}{99} = \frac{610}{33}$$

8. Express $0.12\overline{3}$ in the form $\frac{p}{q}$ where p, q are integers and $q \neq 0$. **(2012)**

Ans. Let $x = 0.12\overline{3}$

Multiplying both the sides by 100,

$$100x = 12.333\ldots \quad \rightarrow(1)$$

Multiplying both the sides by 1000,

$$1000x = 123.333\ldots \quad \rightarrow(2)$$

(2) – (1) $\Rightarrow$ $900x = 111$

$$x = \frac{111}{900} = \frac{37}{300}$$

9. Find two rational numbers in the form $\frac{p}{q}$ between 0.343443444344443 . . . and 0.3636636663 . . . **(2010)**

Ans. Let the rational numbers be 0.35 and 0.36

$$\Rightarrow 0.35 = \frac{35}{100} = \frac{7}{20}$$

and $0.36 = \frac{36}{100} = \frac{9}{25}$

10. How many irrational numbers lie between $\sqrt{2}$ and $\sqrt{3}$? Find any three irrational numbers between $\sqrt{2}$ and $\sqrt{3}$. **(2010)**

Ans. Infinitely many irrational numbers. $\sqrt{2}$ = 1.414 . . . and $\sqrt{3}$ = 1.732 . . .

(i) 1.4040040004 . . . (ii) 1.515115111. . . (iii) 1.606006000 . . .

11. Find three rational numbers between $\frac{3}{5}$ and $\frac{7}{8}$. **(2011)**

Ans. LCM of 5 and 8 = 40, $\frac{3}{5} = \frac{3 \times 8}{5 \times 8} = \frac{24}{40}$

$$\frac{7}{8} = \frac{7 \times 5}{8 \times 5} = \frac{35}{40}$$

$\frac{25}{40}, \frac{26}{40}$ and $\frac{27}{40}$ are the 3 rational number between $\frac{3}{5}$ and $\frac{7}{8}$.

12. Express the number $0.\overline{53}$ in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **(2010, 2012)**

Ans. Let, $x = 0.\overline{53} = 0.5353\ldots \rightarrow (1)$

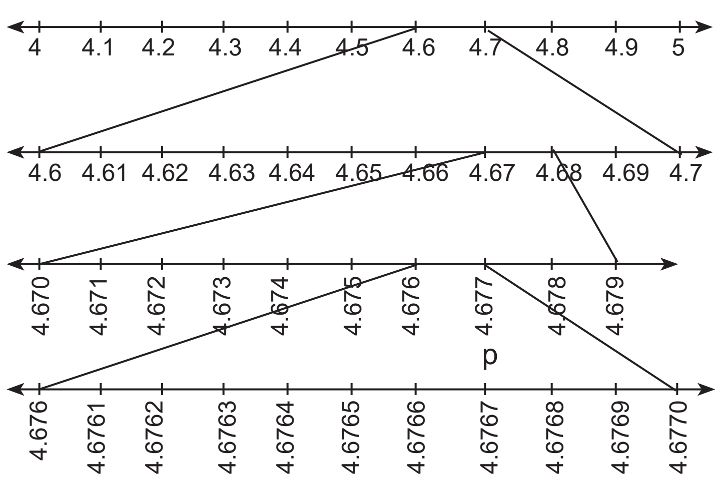
$(1) \times 100 \Rightarrow \quad 100x = 53.5353\ldots \rightarrow (2)$

$(2) - (1) \Rightarrow \quad 99x = 53$

$x = \frac{53}{99}$ which is in the form $\frac{p}{q}$.

13. Visualise the representation of $4.\overline{67}$ on the number line upto 4-decimal places. **(2012)**

Ans. $4.\overline{67} = 4.6767$



The point 'P' represents $4.\overline{67}$.

14. Express $3.42\overline{5}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **(2010, 2012)**

Ans. Let $x = 3.42555...$ $\rightarrow (1)$

(1) x 100 ⇒ $100x = 342.555...$ $\rightarrow (2)$

(1) x 1000 ⇒ $1000x = 3425.555...$ $\rightarrow (3)$

(3) - (2) ⇒ $900x = 3083$

$$x = \frac{3083}{900}$$

EXERCISE 1.2

Topics : Operations on Real Numbers.

2 MARKS

1. Divide $5\sqrt{45}$ by $\frac{\sqrt{75}}{\sqrt{5}}$ **(2015)**

Ans.

$$5\sqrt{45} = 5\sqrt{9 \times 5} = 15\sqrt{5}$$

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

$$\therefore 5\sqrt{45} \div \frac{\sqrt{75}}{\sqrt{5}} = 5\sqrt{45} \times \frac{\sqrt{5}}{\sqrt{75}}$$

$$= \frac{15\sqrt{5} \times \sqrt{5}}{5\sqrt{3}} = \frac{15}{\sqrt{3}} = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3}$$

2. Simplify :$(4\sqrt{5} - 3\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$ **(2014)**

Ans. $(4\sqrt{5} - 3\sqrt{2})(4\sqrt{5} + 3\sqrt{2}) = (4\sqrt{5})^2 - (3\sqrt{2})^2 = 16 \times 5 - 9 \times 2$

$$= 80 - 18 = 62$$

3. Rationalise the denominator of $\frac{1}{\sqrt{7} - \sqrt{6}}$. **(2014)**

Ans. $\frac{1}{\sqrt{7} - \sqrt{6}} = \frac{1}{\sqrt{7} - \sqrt{6}} \times \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} + \sqrt{6}} = \frac{\sqrt{7} + \sqrt{6}}{1} = \sqrt{7} + \sqrt{6}$

4. Rationalize the denominator of $\frac{2}{\sqrt{3} - \sqrt{5}}$. **(2014)**

Ans.

$$\frac{2}{\sqrt{3} - \sqrt{5}} = \frac{2}{\sqrt{3} - \sqrt{5}} \times \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} + \sqrt{5}} = \frac{2(\sqrt{3} + \sqrt{5})}{3 - 5}$$

$$= \frac{2(\sqrt{3} + \sqrt{5})}{-2} = -(\sqrt{3} + \sqrt{5})$$

5. If $\frac{1 + \sqrt{2}}{1 - \sqrt{2}} + \frac{1 - \sqrt{2}}{1 + \sqrt{2}} = a + b\sqrt{2}$, then find a and b. **(2014)**

Ans.

$$\frac{1 + \sqrt{2}}{1 - \sqrt{2}} + \frac{1 - \sqrt{2}}{1 + \sqrt{2}} = \frac{(1 + \sqrt{2})^2 + (1 - \sqrt{2})^2}{1 - 2}$$

$$= \frac{1 + 2 + 2\sqrt{2} + 1 + 2 - 2\sqrt{2}}{-1} = -6$$

i.e., $-6 + 0\sqrt{2} = a + b\sqrt{2} \Rightarrow a = -6, b = 0$

6. If $a = 3 - 2\sqrt{2}$, then find the value of $a^2 - \frac{1}{a^2}$. **(2014)**

Ans. $\frac{1}{a} = \frac{1}{3 - 2\sqrt{2}} \times \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}} = \frac{3 + 2\sqrt{2}}{9 - 8} = 3 + 2\sqrt{2}$

$\therefore a^2 - \frac{1}{a^2} = (3 - 2\sqrt{2})^2 - (3 + 2\sqrt{2})^2$

$= 9 - 12\sqrt{2} + 8 - (9 + 12\sqrt{2} + 8) = -24\sqrt{2}$

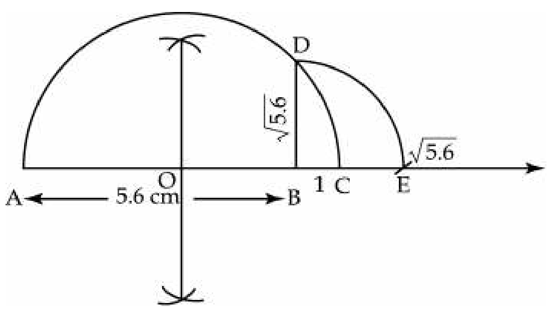
7. Simplify: $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ **(2013)**

Ans. $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = \sqrt{9 \times 5} - 3\sqrt{5 \times 4} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} = \sqrt{5}$

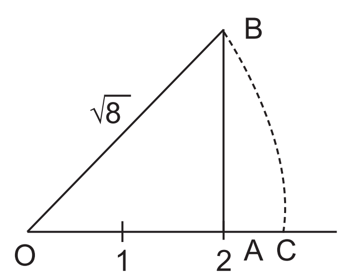
8. Represent geometrically $\sqrt{5.6}$ on the number line. **(2012, 2013)**

Ans.



9. Represent $\sqrt{8}$ on the number line. **(2012)**

Ans. Mark OA = 2 unit and AB = 2 unit in the number line as shown.



Then $OB^2 = OA^2 + AB^2$

$= 2^2 + 2^2 = 8$

$\therefore OB = \sqrt{8}$

With O as centre and OB as radius draw an arc to cut the number line at C.

Then $OC = \sqrt{8}$

10. If $\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = a + b\sqrt{2}$, then find the value of a and b. **(2011)**

Ans. $\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{(\sqrt{2} - 1)^2}{(\sqrt{2})^2 - 1^2}$

$= \frac{2 - 2\sqrt{2} + 1}{2 - 1} = \frac{3 - 2\sqrt{2}}{1} = 3 - 2\sqrt{2}$

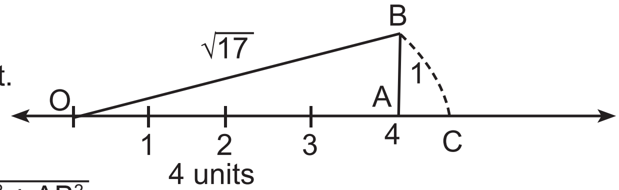
But, $3 - 2\sqrt{2} = a + b\sqrt{2}$

$\therefore a = 3, b = -2$

11. Represent $\sqrt{17}$ on the number line. **(2011)**

Ans. Draw AB⊥OA.

Such that OA = 4 unit and AB = 1 unit.



By Pythagoras Theorem, $OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{16 + 1} = \sqrt{17}$

Now with O as centre and OB as radius draw an arc to intersect the line at C. Then C represents $\sqrt{17}$.

3 MARKS

12. If $x = 4 - \sqrt{15}$, find the value of $(x + \frac{1}{x})^2$. **(2015)**

Ans. $\frac{1}{x} = \frac{1}{4 - \sqrt{15}} \times \frac{4 + \sqrt{15}}{4 + \sqrt{15}} = \frac{4 + \sqrt{15}}{16 - 15} = 4 + \sqrt{15}$

$\therefore (x + \frac{1}{x})^2 = (4 - \sqrt{15} + 4 + \sqrt{15})^2 = (8)^2 = 64$

13. Simplify: $\sqrt{\frac{\sqrt{20} + \sqrt{11}}{\sqrt{20} - \sqrt{11}}}$ **(2014)**

Ans. $\sqrt{\frac{\sqrt{20} + \sqrt{11}}{\sqrt{20} - \sqrt{11}}} = \frac{\sqrt{\sqrt{20} + \sqrt{11}}}{\sqrt{\sqrt{20} - \sqrt{11}}} \times \frac{\sqrt{\sqrt{20} + \sqrt{11}}}{\sqrt{\sqrt{20} + \sqrt{11}}}$

$= \frac{(\sqrt{\sqrt{20} + \sqrt{11}})^2}{\sqrt{(\sqrt{20})^2 - (\sqrt{11})^2}} = \frac{\sqrt{20} + \sqrt{11}}{\sqrt{20 - 11}} = \frac{\sqrt{20} + \sqrt{11}}{3}$

14. Simplify : $\sqrt{2}(\sqrt{6} - \sqrt{18}) + \sqrt{3}(\sqrt{27} - \sqrt{6}) + 3\sqrt{2}$ **(2012)**

Ans. $\sqrt{2}(\sqrt{6} - \sqrt{18}) + \sqrt{3}(\sqrt{27} - \sqrt{6}) + 3\sqrt{2} = \sqrt{12} - \sqrt{36} + \sqrt{81} - \sqrt{18} + 3\sqrt{2}$

$= 2\sqrt{3} - 6 + 9 - 3\sqrt{2} + 3\sqrt{2} = 2\sqrt{3} + 3$

15. Simplify the following into a fraction with rational denominator. $\frac{1}{\sqrt{5} + \sqrt{6} - \sqrt{11}}$ **(2012)**

Ans. $\frac{1}{(\sqrt{5} + \sqrt{6}) - \sqrt{11}} \times \frac{(\sqrt{5} + \sqrt{6}) + \sqrt{11}}{(\sqrt{5} + \sqrt{6}) + \sqrt{11}} = \frac{\sqrt{5} + \sqrt{6} + \sqrt{11}}{(\sqrt{5} + \sqrt{6})^2 - (\sqrt{11})^2}$

$= \frac{\sqrt{5} + \sqrt{6} + \sqrt{11}}{5 + 6 + 2\sqrt{5} \times \sqrt{6} - 11} = \frac{\sqrt{5} + \sqrt{6} + \sqrt{11}}{2\sqrt{30}}$

$= \frac{\sqrt{5} + \sqrt{6} + \sqrt{11}}{2\sqrt{30}} \times \frac{\sqrt{30}}{\sqrt{30}}$

$= \frac{(\sqrt{5} + \sqrt{6} + \sqrt{11})\sqrt{30}}{2 \times 30} = \frac{(\sqrt{5} + \sqrt{6} + \sqrt{11})\sqrt{6} \times \sqrt{5}}{60}$

$$= \frac{5\sqrt{6} + 6\sqrt{5} + \sqrt{330}}{60}$$

16. If $a = 9 - 4\sqrt{5}$, find the value of $a^2 + \frac{1}{a^2}$. **(2011, 2012, 2013)**

Ans.

$$a = 9 - 4\sqrt{5},$$

$$\frac{1}{a} = \frac{1}{9 - 4\sqrt{5}} \times \frac{9 + 4\sqrt{5}}{9 + 4\sqrt{5}}$$

$$= \frac{9 + 4\sqrt{5}}{9^2 - (4\sqrt{5})^2} = \frac{9 + 4\sqrt{5}}{1} = 9 + 4\sqrt{5}$$

$$\therefore a + \frac{1}{a} = 9 - 4\sqrt{5} + 9 + 4\sqrt{5} = 18 \quad \rightarrow(i)$$

$$\text{Now } \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2a \cdot \frac{1}{a} \quad [\because (a + b)^2 = a^2 + b^2 + 2ab]$$

$$18^2 = a^2 + \frac{1}{a^2} + 2 \quad \text{[From (i)]}$$

$$324 - 2 = a^2 + \frac{1}{a^2}$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 322$$

17. Simplify: $\frac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6} - \sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6} + 2}$ **(2012)**

Ans. $\frac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}} \times \frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} + \sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6} - \sqrt{2}} \times \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6} + 2} \times \frac{\sqrt{6} - 2}{\sqrt{6} - 2}$

$$= \frac{3\sqrt{12} + 3\sqrt{6}}{3} - \frac{4\sqrt{18} + 4\sqrt{6}}{4} + \frac{2\sqrt{18} - 4\sqrt{3}}{2}$$

$$= \sqrt{12} + \cancel{\sqrt{6}} - \cancel{\sqrt{18}} - \cancel{\sqrt{6}} + \cancel{\sqrt{18}} - 2\sqrt{3} = 2\sqrt{3} - 2\sqrt{3} = 0$$

18. Rationalise the denominator and hence find the value of $\frac{6}{\sqrt{5} + \sqrt{3}}$ if $\sqrt{5} = 2.236$ and $\sqrt{3} = 1.732$ **(2011)**

Ans.

$$\frac{6}{\sqrt{5} + \sqrt{3}} = \frac{6}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{6(\sqrt{5} - \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{6(\sqrt{5} - \sqrt{3})}{2} = 3(\sqrt{5} - \sqrt{3})$$

$$= 3(2.236 - 1.732) = 3(0.504) = 1.512$$

4 MARKS

19. Show that : $\frac{1}{3 - \sqrt{8}} - \frac{1}{\sqrt{8} - \sqrt{7}} + \frac{1}{\sqrt{7} - \sqrt{6}} - \frac{1}{\sqrt{6} - \sqrt{5}} + \frac{1}{\sqrt{5} - 2} = 5$ **(2014)**

Ans. LHS $= \dfrac{1}{3-\sqrt{8}} \times \dfrac{3+\sqrt{8}}{3+\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} \times \dfrac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} \times \dfrac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$- \dfrac{1}{\sqrt{6}-\sqrt{5}} \times \dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} + \dfrac{1}{\sqrt{5}-2} \times \dfrac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \dfrac{3+\sqrt{8}}{1} - \dfrac{\sqrt{8}+\sqrt{7}}{1} + \dfrac{\sqrt{7}+\sqrt{6}}{1} - \dfrac{\sqrt{6}+\sqrt{5}}{1} + \dfrac{\sqrt{5}+2}{1}$

$= 3 + \sqrt{8} - \sqrt{8} - \sqrt{7} + \sqrt{7} + \sqrt{6} - \sqrt{6} - \sqrt{5} + \sqrt{5} + 2 = 5 =$ RHS

20. Simplify : $\dfrac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \dfrac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \dfrac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$. **(2011, 2013)**

Ans. $\dfrac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \dfrac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \dfrac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

$= \dfrac{2\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} + \dfrac{6\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})} - \dfrac{8\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$

$= \dfrac{2\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3} + \dfrac{6\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3} - \dfrac{8\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2}$

$= -(2\sqrt{12} - 2\sqrt{18}) + \dfrac{\overset{2}{\cancel{6}}\sqrt{2}(\sqrt{6}-\sqrt{3})}{\cancel{3}} - \dfrac{\overset{2}{\cancel{8}}\sqrt{3}(\sqrt{6}-\sqrt{2})}{\cancel{4}}$

$= -2\sqrt{12} + 2\sqrt{18} + 2\sqrt{12} - 2\sqrt{6} - 2\sqrt{18} + 2\sqrt{6} = 0$

EXERCISE 1.3

Topics : Laws of Exponents for Real Numbers.

2 MARKS

1. Simplify : $\dfrac{2^{p-q} . 2^{r-p}}{2^{r-q}}$ **(2016)**

Ans. $\dfrac{2^{(p-q+r-p)}}{2^{r-q}} = \dfrac{2^{(r-q)}}{2^{(r-q)}} = 1$

2. Find the value of x, if $\sqrt[5]{5x+2} = 2$ **(2015)**

Ans. Given, $\sqrt[5]{5x+2} = 2$

$\Rightarrow (5x+2)^{\frac{1}{5}} = 2$

$\Rightarrow [(5x+2)^{\frac{1}{5}}]^5 = 2^5$

$5x + 2 = 32$

$\Rightarrow 5x = 30$

$x = 6$

3. Simplify : $\left(\frac{64}{125}\right)^{-2/3}$ **(2010, 2011, 2014)**

Ans. $\left(\frac{64}{125}\right)^{-2/3} = \left(\frac{125}{64}\right)^{2/3} = \left(\frac{5}{4}\right)^{3 \times 2/3} = \left(\frac{5}{4}\right)^{2} = \frac{25}{16}$

4. If $125^x = \frac{25}{5^x}$, find x. **(2014)**

Ans.

$$125^x = \frac{25}{5^x}$$
$$125^x \times 5^x = 25$$
$$(125 \times 5)^x = 25$$
$$(25^2)^x = 25$$
$$(25)^{2x} = (25)^1$$
$$\Rightarrow 2x = 1 \qquad \Rightarrow x = \frac{1}{2}$$

5. Find the value of x if $\sqrt[3]{3x - 2} = 4$. **(2016, 2013)**

Ans.

$$\text{Given, } \sqrt[3]{3x - 2} = 4$$
$$\text{i.e., } (3x - 2)^{1/3} = 4$$
$$\text{i.e., } [(3x - 2)^{1/3}]^3 = (4)^3$$
$$3x - 2 = 64$$
$$3x = 66$$
$$x = 22$$

6. Simplify $\sqrt[4]{\sqrt[3]{x^2}}$ and express the result in the exponential form of x. **(2015)**

Ans. $\sqrt[4]{\sqrt[3]{x^2}} = \left((x^2)^{\frac{1}{3}}\right)^{\frac{1}{4}} = x^{\left(2 \times \frac{1}{3} \times \frac{1}{4}\right)} = x^{\frac{1}{6}}$

7. Simplify $\sqrt[4]{81} - 8 . \sqrt[3]{216} + 15 . \sqrt[5]{32} + \sqrt{225}$ **(2012, 2013)**

Ans. $\sqrt[4]{81} - 8 . \sqrt[3]{216} + 15 . \sqrt[5]{32} + \sqrt{225} = \sqrt[4]{3^4} - 8 \times \sqrt[3]{6^3} + 15 \times \sqrt[5]{2^5} + 15$

$= 3^{4/4} - 8 \times 6^{3/3} + 15 \times 2^{5/5} + 15 = 3 - 48 + 30 + 15 = 0$

8. Evaluate: $\left(\frac{32}{243}\right)^{\frac{-4}{5}}$ **(2011)**

Ans. $\left(\frac{32}{243}\right)^{\frac{-4}{5}} = \left(\frac{243}{32}\right)^{\frac{4}{5}} = \left(\frac{3}{2}\right)^{5 \times \frac{4}{5}} = \left(\frac{3}{2}\right)^{4} = \frac{81}{16}$

9. Simplify $\left(\frac{64}{25}\right)^{\frac{-3}{2}}$ **(2011)**

Ans. $\left(\frac{64}{25}\right)^{\frac{-3}{2}} = \left(\frac{25}{64}\right)^{\frac{3}{2}} = \left(\frac{5}{8}\right)^{2 \times \frac{3}{2}} = \left(\frac{5}{8}\right)^{3} = \frac{125}{512}$

3 MARKS

10. Show that $\left(\frac{x^{a(b-c)}}{x^{b(a-c)}}\right) \div \left[\frac{x^{b}}{x^{a}}\right]^{c} = 1.$ **(2014, 2013)**

Ans. $\left(\frac{x^{a(b-c)}}{x^{b(a-c)}}\right) \div \left[\frac{x^{b}}{x^{a}}\right]^{c} = \frac{x^{ab-ac}}{x^{ab-bc}} \div \frac{x^{bc}}{x^{ac}} = x^{ab-ac-(ab-bc)} \times \frac{x^{ac}}{x^{bc}}$

$= x^{ab-ac-ab+bc} \times x^{ac-bc} = x^{ab-ac-ab+bc+ac-bc} = x^{0} = 1$

11. Find the value of x if $\left(\frac{3}{4}\right)^{3}\left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x}$ **(2010)**

Ans. Given, $\left(\frac{3}{4}\right)^{3}\left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x}$ $\left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}\right]$

i.e., $\left(\frac{3}{4}\right)^{3} \times \left(\frac{3}{4}\right)^{7} = \left(\frac{3}{4}\right)^{2x}$ $\left[a^{m} \times a^{n} = a^{m+n}\right]$

$\left(\frac{3}{4}\right)^{10} = \left(\frac{3}{4}\right)^{2x}$

$\Rightarrow 10 = 2x$

$x = \frac{10}{2} = 5$

12. Simplify : $\left(\frac{3^{-1} \times 5^{2}}{3^{2} \times 5^{-4}}\right)^{\frac{1}{3}} \times \left(\frac{3^{-1} \times 5^{-1}}{3^{3} \times 5^{-5}}\right)^{\frac{-1}{2}}.$ **(2010, 2011)**

Ans. $\left(\frac{3^{-1} \times 5^{2}}{3^{2} \times 5^{-4}}\right)^{\frac{1}{3}} \times \left(\frac{3^{-1} \times 5^{-1}}{3^{3} \times 5^{-5}}\right)^{\frac{-1}{2}} = \left(\frac{5^{6}}{3^{3}}\right)^{\frac{1}{3}} \times \left(\frac{5^{4}}{3^{4}}\right)^{\frac{-1}{2}}$

$= \left(\frac{5^{2}}{3}\right)^{3 \times \frac{1}{3}} \times \left(\frac{5}{3}\right)^{4 \times \frac{-1}{2}}$

$= \frac{5^{2}}{3} \times \left(\frac{5}{3}\right)^{-2} = \frac{5^{2}}{3} \times \left(\frac{3}{5}\right)^{2} = \frac{5^{2}}{3} \times \frac{3^{2}}{5^{2}} = 3$

13. If a = 2, b = 3 then find the values of the following (i) $(a^{b} + b^{a})^{-1}$ (ii) $(a^{a} + b^{b})^{-1}$ **(2010, 2012)**

Ans. (i) $(a^{b} + b^{a})^{-1} = (2^{3} + 3^{2})^{-1} = (8 + 9)^{-1} = 17^{-1} = \frac{1}{17}$

(ii) $(a^{a} + b^{b})^{-1} = (2^{2} + 3^{3})^{-1} = (4 + 27)^{-1} = 31^{-1} = \frac{1}{31}$

4 MARKS

14. Show that $\frac{x^{-1} + y^{-1}}{x^{-1}} + \frac{x^{-1} - y^{-1}}{y^{-1}} = \frac{x^{2} + y^{2}}{xy}$ **(2015)**

Ans. LHS $= \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x}} + \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{y}}$

$= \frac{y + x}{xy} \times \frac{x}{1} + \frac{y - x}{xy} \times \frac{y}{1}$

$$= \frac{y + x}{y} + \frac{y - x}{x}$$

$$= \frac{xy + x^2 + y^2 - xy}{xy} = \frac{x^2 + y^2}{xy} = \text{RHS}$$

15. If $x^a = y$, $y^b = z$, $z^c = x$, then prove that abc = 1 **(2016, 2014)**

Ans. Given, $x^a = y \rightarrow (1)$

$y^b = z \rightarrow (2)$

$z^c = x \rightarrow (3)$

(3) $\Rightarrow$ $(y^b)^c = x$ [From (2)]

$y^{bc} = x$

i.e., $(x^a)^{bc} = x$ [From (1)]

i.e., $x^{abc} = x^1$

$\Rightarrow$ abc = 1

16. Prove that $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} . \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} . \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$ **(2012, 2013)**

Ans. LHS $= \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} . \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} . \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$

$$= \left(x^{a - b}\right)^{\frac{1}{ab}} . \left(x^{b - c}\right)^{\frac{1}{bc}} . \left(x^{c - a}\right)^{\frac{1}{ca}}$$

$$= x^{\frac{a-b}{ab}} . x^{\frac{b-c}{bc}} . x^{\frac{c-a}{ca}}$$

$$= x^{\frac{c(a - b) + a(b - c) + b (c - a)}{abc}}$$

$$= x^{\frac{ac - bc + ab - ac + bc - ab}{abc}} = x^{o} = 1 = \text{RHS}$$

1 MARK

1. Identify a rational number among the following numbers : $2 + \sqrt{2}$, $2\sqrt{2}$, 0, π **(2015)**

Ans. Rational number is 0.

2. If $x^{\frac{1}{12}} = 49^{\frac{1}{24}}$, then find the value of x. **(2015)**

Ans. $$x^{\frac{1}{12}} = 7^{2^{\frac{1}{24}}}$$

i.e., $x^{\frac{1}{12}} = 7^{\frac{1}{12}}$

$\Rightarrow x = 7$

3. Find the value of $\frac{3^o + 5^o}{4^o}$. **(2016, 2014)**

Ans. $\frac{3^o + 5^o}{4^o} = \frac{1 + 1}{1} = 2$

4. Write the rationalising factor of $\frac{1}{\sqrt{7} - \sqrt{4}}$ **(2015)**

Ans. The rationalising factor is $\sqrt{7} + \sqrt{4}$.

5. If p and q are integers with q ≠ 0, then express 121.111… in $\frac{p}{q}$ form. **(2015)**

Ans.

Let x = 121.11...... →(1)

So, $10x$ = 1211.11 ...... →(2)

(2) - (1) ⇒ $9x = 1090$

$\Rightarrow x = \frac{1090}{9}$

6. Write the rationalising factor of the denominator in $\frac{1}{\sqrt{2} + \sqrt{3}}$ **(2014)**

Ans. $\sqrt{2} - \sqrt{3}$

7. Find the value of $(256)^{0.16} \times (256)^{0.09}$. **(2013)**

Ans. $(256)^{0.16} \times (256)^{0.09} = (256)^{0.16 + 0.09} = (256)^{0.25}$

$= (256)^{1/4} = (4^4)^{1/4} = 4$

8. Identify an irrational number among the following numbers:

$0.13, 0.131\overline{5}, 0.\overline{1315}, 0.3013001300013 .....$ **(2014)**

Ans. 0.3013001300013 ....

9. Find one irrational number between 2013 and 2014. **(2013, 2014)**

Ans. 2013.1010010001 ......

10. Find the decimal expansion of $\frac{58}{1000}$. **(2013, 2014)**

Ans. $\frac{58}{1000} = 0.058$

✍ SELF ASSESSMENT TEST

1. Simplify: $[7(81^{1/4} + 256^{1/4})^{1/4}]^4$.

2. Simplify: $\sqrt{72} + \sqrt{800} - \sqrt{18}$.

3. Write the rationalising factor of the denominator in $\frac{1}{\sqrt{2} + \sqrt{3}}$

4. Find the sum of $0.\overline{3}$ and $0.\overline{2}$.

5. Express $2.\overline{36} + 0.\overline{23}$ as a fraction in simplest form.

6. If $p = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$ and $q = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$, find $p^2 + q^2$.

7. If $a = \frac{3 - \sqrt{5}}{3 + \sqrt{5}}$ and $b = \frac{3 + \sqrt{5}}{3 - \sqrt{5}}$, find $a^2 - b^2$.

8. Prove that $\frac{1}{3 + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{3} + 1} = 1$.

9. Simplify: $\left(\frac{81}{16}\right)^{-3/4} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$

10. Write the following in the ascending order of their magnitude : $\sqrt{3}$, $\sqrt[3]{4}$, $\sqrt[4]{6}$

11. Prove that : $\left(\frac{x^a}{x^b}\right)^{a^2 + ab + b^2} . \left(\frac{x^b}{x^c}\right)^{b^2 + bc + c^2} . \left(\frac{x^c}{x^a}\right)^{c^2 + ac + a^2} = 1$.

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