SELF ASSESSMENT TEST SOLUTIONS

1. Here, a = 2, b = - k and c = k .

   For equal roots, $b^2 - 4ac = 0$

$$(-k)^2 - 4(2)k = 0$$

$$k^2 - 8k = 0$$

$$k(k - 8) = 0 \quad \Rightarrow k = 0, 8$$

   So (d) is the correct option.

2. Here, a = 1, b = k and c = 1.

   For linear factors, $b^2 - 4ac \geq 0$

$$k^2 - 4 \times 1 \times 1 \geq 0$$

$$k^2 - 2^2 \geq 0$$

$$(k - 2)(k + 2) \geq 0$$

$$k \geq 2 \text{ and } k \leq -2$$

   So (d) is the correct option.

3. If one root is α, then the other root is $\frac{1}{\alpha}$

   Product of roots, $\alpha . \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{c}{a} \Rightarrow a = c$$

   So (d) is the correct option.

4.

$$(x^2 + 1)^2 - x^2 = 0$$

$$x^4 + 1 + 2x^2 - x^2 = 0$$

$$x^4 + x^2 + 1 = 0$$

$$(x^2)^2 + x^2 + 1 = 0$$

   Let $x^2 = y$ then, $y^2 + y + 1 = 0$

   Here, a = 1, b = 1 and c = 1

$$D = b^2 - 4ac$$

$$= (1)^2 - 4(1)(1) = 1 - 4 = -3$$

   Since, D < 0, $y^2 + y + 1 = 0$ has no real roots.

   i.e. $(x^2 + 1)^2 - x^2 = 0$ has no real roots.

   So (c) is the correct option.

5. Given $x^2 + y^2 = 25$

$$x^2 + \left(\frac{12}{x}\right)^2 = 25 \qquad [\text{Since } xy = 12]$$

$$x^4 + 144 - 25x^2 = 0$$

$$(x^2 - 16)(x^2 - 9) = 0$$

ie. $x^2 = 16 \Rightarrow x = \pm 4$

and $x^2 = 9 \Rightarrow x = \pm 3$

So (c) is the correct option.

6. $$4\sqrt{3}\, x^2 + 5x - 2\sqrt{3} = 0$$

$$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$(\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$$

$\Rightarrow \quad x = -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

7. $$(x + 3)(x - 1) = 3\left(x - \frac{1}{3}\right)$$

$$x^2 + 3x - x - 3 = 3x - 1$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x - 2) + 1(x - 2) = 0$$

$$(x - 2)(x + 1) = 0$$

$\Rightarrow x = 2, -1$

8. $$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$$

$$\frac{2x^2 - 5x - 3}{5} = 0$$

$$2x^2 - 5x - 3 = 0$$

$$2x^2 - 6x + x - 3 = 0$$

$$2x(x - 3) + 1(x - 3) = 0$$

$$(2x + 1)(x - 3) = 0$$

$\Rightarrow x = -\frac{1}{2}, 3$

9. $$x^2 - 2ax - (4b^2 - a^2) = 0$$

$$x^2 - 2ax + a^2 - 4b^2 = 0$$

$$(x - a)^2 - (2b)^2 = 0$$

$$(x - a + 2b)(x - a - 2b) = 0$$

$\Rightarrow x = a - 2b, x = a + 2b$

10. Here, $a = 13\sqrt{3}$, $b = 10$, $c = \sqrt{3}$

$$b^2 - 4ac = (10)^2 - 4(13\sqrt{3})(\sqrt{3})$$
$$= 100 - 156 = -56$$

As D < 0, the equation has no real roots.

11. $$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$$

$$2x(2x+3) + 1(x-3) + (3x+9) = 0$$

$$4x^2 + 6x + x - 3 + 3x + 9 = 0$$

$$4x^2 + 10x + 6 = 0$$

$$2x^2 + 5x + 3 = 0$$

$$(x+1)(2x+3) = 0$$

$\Rightarrow x = -1, x = -\frac{3}{2}$

12. $$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$$

$$x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$$

$$x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$$

$$\left(x + \frac{a}{a+b}\right)\left(x + \frac{a+b}{a}\right) = 0$$

$\Rightarrow x = \frac{-a}{a+b}, \frac{-(a+b)}{a}$

13. Put $\frac{2x}{x-5} = y$ then, $y^2 + 5y - 24 = 0$

$$(y+8)(y-3) = 0$$

$$\Rightarrow y = 3, -8$$

Put $y = 3$. Then we have $\frac{2x}{x-5} = 3$

$$\Rightarrow 2x = 3x - 15 \Rightarrow x = 15$$

Put $y = -8$. Then we have $\frac{2x}{x-5} = -8$

$$2x = -8x + 40$$

$$10x = 40 \Rightarrow x = 4$$

ie. $x = 15, 4$

SELF ASSESSMENT TEST SOLUTIONS

14. Now $2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$

$= 2a[a + 2b] + b[a + 2b]$

$= (a + 2b)(2a + b)$

Given, $9x^2 - 9(a + b)x + 2a^2 + 5ab + 2b^2 = 0$

ie. $9x^2 - 9(a + b)x + (a + 2b)(2a + b) = 0$

$9x^2 - 3[3a + 3b]x + (a + 2b)(2a + b) = 0$

$9x^2 - 3[(a + 2b) + (2a + b)]x + (a + 2b)(2a + b) = 0$

$9x^2 - 3(a + 2b)x - 3(2a + b)x + (a + 2b)(2a + b) = 0$

$3x[3x - (a + 2b)] - (2a + b)[3x - (a + 2b)] = 0$

$[3x - (a + 2b)][3x - (2a + b)] = 0$

ie. $3x - (2a + b) = 0$

$\Rightarrow x = \frac{a + 2b}{3}$

or $3x - (2a + b) = 0$

$\Rightarrow x = \frac{2a + b}{3}$

So roots are $\frac{a + 2b}{3}$ and $\frac{2a + b}{3}$.

15. $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$

$x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$

$x^2b^2 + y^2a^2 - 2abxy = 0$

$(xb - ya)^2 = 0$

$xb = ya$

$\Rightarrow \frac{x}{a} = \frac{y}{b}$