

TEACHERS FORUM[®]



QUESTION BANK

(solved)

Based on CBSE previous years' question papers

Class X

MATHEMATICS

SUBJECT EXPERTS

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REAL NUMBERS

EXERCISE 1.1

Topics :

- ◆ The Fundamental Theorem of Arithmetic : Every composite number can be expressed as product of primes and this factorisation is unique, apart from the order in which the prime factors occur.
- ◆ For any two positive integers a and b, $HCF(a, b) \times LCM(a, b) = a \times b$

2 MARKS

1. Find HCF of 44, 96 and 404 by prime factorization method. Hence find their LCM.

Ans. $44 = 2 \times 2 \times 11 = 2^2 \times 11$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$404 = 2 \times 2 \times 101 = 2^2 \times 101$

$HCF = 2^2 = 4$

$LCM = 2^5 \times 11 \times 3 \times 101 = 106656$

$\begin{array}{r} 2 \overline{) 44} \\ 2 \overline{) 22} \\ \hline 11 \end{array}$	$\begin{array}{r} 2 \overline{) 96} \\ 2 \overline{) 48} \\ 2 \overline{) 24} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ \hline 3 \end{array}$	$\begin{array}{r} 2 \overline{) 404} \\ 2 \overline{) 202} \\ \hline 101 \end{array}$	(2020 - B)
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2. Check whether 6^n can end with the digit '0' (zero) for any natural number n. **(2020-B)**

Ans. $6^n = (2 \times 3)^n = 2^n \times 3^n$

It is not in form of $2^n \times 5^m$

$\therefore 6^n$ can't end with digit '0'

3. Find the LCM of 150 and 200. **(2020-B)**

Ans. $150 = 2 \times 3 \times 5^2$

$200 = 2^3 \times 5^2$

$LCM = 2^3 \times 5^2 \times 3 = 600$

$\begin{array}{r} 5 \overline{) 150} \\ 5 \overline{) 30} \\ 3 \overline{) 6} \\ \hline 2 \end{array}$	$\begin{array}{r} 5 \overline{) 200} \\ 5 \overline{) 40} \\ 2 \overline{) 8} \\ 2 \overline{) 4} \\ \hline 2 \end{array}$
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4. The HCF of two numbers is 116 and their LCM is 1740. If one number is 580, find the other. **(2019)**

Ans. $a \times b = LCM \times HCF$

\therefore Other number = $\frac{116 \times 1740}{580} = 348$

5. Find the HCF of 612 and 1314 using prime factorisation. **(2019)**

Ans. $612 = 2^2 \times 3^2 \times 17$

$1314 = 2 \times 3^2 \times 73$

$\therefore HCF(612, 1314) = 2 \times 3^2 = 18$

$\begin{array}{r} 2 \overline{) 612} \\ 2 \overline{) 306} \\ 3 \overline{) 153} \\ 3 \overline{) 51} \\ \hline 17 \end{array}$	$\begin{array}{r} 2 \overline{) 1314} \\ 3 \overline{) 657} \\ 3 \overline{) 219} \\ \hline 73 \end{array}$
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6. Write the smallest number which is divisible by both 306 and 657. **(2019)**

Ans. Smallest number divisible by 306 and 657 = LCM (306, 657)

$$\begin{array}{r} 3 \overline{) 306, 657} \\ 3 \overline{) 102, 219} \\ \quad 34, 73 \end{array}$$

\therefore LCM (306, 657) = $3 \times 3 \times 34 \times 73 = 22338$

7. The HCF of two numbers is 145 and the product of the two numbers is 315375. Find the LCM of the two numbers. **(2019)**

Ans. LCM \times HCF = $a \times b$

$$\text{LCM} = \frac{315375}{145} = 2175$$

8. If HCF of 65 and 117 is expressible in the form $65n - 117$, then find the value of n .

Ans. HCF (65, 117) = 13 **(2019)**

$$\text{But, } 13 = 65n - 117$$

$$\Rightarrow 65n = 130$$

$$\Rightarrow n = 2$$

$$\begin{array}{r} 13 \overline{) 65, 117} \\ \quad 5, 9 \end{array}$$

9. On a morning walk, three persons step out together and their steps measure 30 cm, 36 cm and 40 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? **(2019)**

Ans. Required minimum distance = LCM (30, 36, 40)

$$30 = 2 \times 3 \times 5$$

$$36 = 2^2 \times 3^2$$

$$40 = 2^3 \times 5$$

$$\therefore \text{Minimum distance} = 2^3 \times 3^2 \times 5 = 360 \text{ cm}$$

10. The LCM of 2 numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number. **(2013, 2014)**

Ans. LCM = 14 HCF \rightarrow (1)

$$\text{LCM} + \text{HCF} = 600 \quad \rightarrow$$
(2)

$$\Rightarrow 15 \text{ HCF} = 600 \quad [\text{From (1)}]$$

$$\Rightarrow \text{HCF} = 40$$

$$(2) \Rightarrow \text{LCM} = 560$$

We know that, LCM \times HCF = Product of 2 nos

$$560 \times 40 = 280 \times x$$

$$80 = x$$

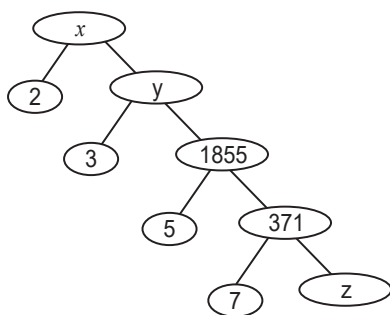
$$\therefore \text{Other number} = 80$$

Similar Problems for Practice

1. Find HCF of 105 and 1515 by prime factorisation method. Hence find their LCM also. **(2013)**
2. Given that $HCF(306, 1314) = 18$, find $LCM(306, 1314)$. **(2012, 2013)**
3. Find the HCF and LCM of 404 and 96 and verify $HCF \times LCM = \text{product of two given numbers}$. **(2013)**
4. Find the L.C.M. of 24, 60, 150 by fundamental theorem of arithmetic. **(2012, 2013)**

Ans. 1. H.C.F = 15, L.C.M. = 10605 2. 22338 3. 4, 9696 4. 600

11. Complete the following factor tree and find the composite number x . **(2015)**



Ans.

$$z = \frac{371}{7} = 53$$

$$y = 1855 \times 3 = 5565$$

$$x = 5565 \times 2 = 11130$$

12. State Fundamental theorem of Arithmetic. Hence find the number of divisors of 1024.

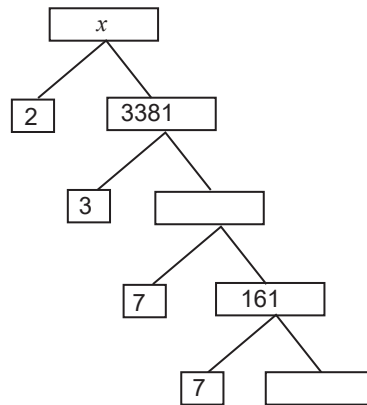
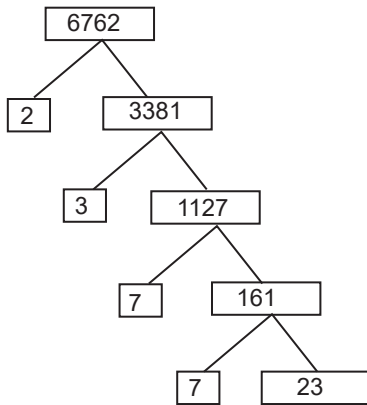
Ans. Every composite number can be expressed as product of primes and this factorisation is unique, apart from the order in which the prime factors occur. **(2014)**

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4

$$2^{10} \quad 1024 = (2)^{10}$$

13. Complete the following factor tree and find the composite number x . **(2014)**

Ans.



Composite number, $x = 6762$

14. Can two numbers have 15 as their HCF and 175 as their LCM? Give reasons.

Ans. No.

(2010, 2012)

15 does not divide 175.

LCM is exactly divisible by their HCF.

15. Show that 9^n can't end with 2 for any integer n.

(2010)

Ans. Let us assume 9^n ends with digit 2 $\Rightarrow 9^n$ is an even number.

$\Rightarrow 2$ is a factor of 9^n . This is not possible because $9^n = (3^2)^n$. So by the uniqueness of Fundamental Theorem of Arithmetic, 2 is not a factor of 9^n . So our assumption is wrong and 9^n will not end with digit 2.

16. Show that the number 4^n , where n is a natural number cannot end with the digit zero for any natural number, n.

(2011, 2012, 2013)

Ans. If the number 4^n , were to end with the digit zero, then it should be divisible by 5.

But $4^n = 2^{2n}$

\Rightarrow Only prime in the factorization of 4^n is 2.

So by fundamental theorem of Arithmetic, there are no other primes in the factorisation of 4^n .

$\Rightarrow 4^n$ can never end with the digit zero.

17. Three bells toll at intervals of 9, 12, 15 minute respectively. If they start tolling together, after what time will they next toll together.

(2013)

Real Numbers

Ans. 3 $\left\{ \begin{array}{l} 9, 12, 15 \\ 3, 4, 5 \end{array} \right.$ $\text{LCM}(9, 12, 15) = 3 \times 3 \times 4 \times 5 = 180$ minutes
 i.e., the bells will toll together after 180 minutes

18. Find the greatest number of 6 digit exactly divisible by 24,15 and 36. **(2013)**

Ans. $\text{LCM of } (24, 15, 36) = 2 \times 3 \times 2 \times 2 \times 5 \times 3 = 360$

The greatest six digit number is 999999.

When 999999 divided by 360, the quotient is 2777 and remainder is 279.

The required number = $999999 - 279 = 999720$

2	24,15, 36
3	12,15,18
2	4, 5, 6
	2, 5, 3

19. Show that the numbers 143 and 187 are not co-prime. **(2015)**

Ans. $143 = 11 \times 13, \quad 187 = 11 \times 17$

Since 143 and 187 can be expressed as product of primes these are not co-primes.

20. Find HCF of two numbers whose prime factorisation are expressible as $2^3 \times 5^2 \times 7 \times 13$ and $2^3 \times 5$. **(2015)**

Ans. $\text{HCF} = 2^3 \times 5 = 40$

21. Find LCM of numbers whose prime factorisation are expressible as 3×5^2 and $3^2 \times 7^2$.

Ans. $\therefore \text{LCM} = 3 \times 5^2 \times 3 \times 7^2 = 3^2 \times 5^2 \times 7^2$

$$= (3 \times 5 \times 7)^2$$

$$= (105)^2 = 11025$$

3	$3 \times 5^2, 3^2 \times 7^2$
	$5^2, 3 \times 7^2$

(2014)

22. Write the prime factorization of 27300. **(2014)**

Ans. $27300 = 2^2 \times 3 \times 5^2 \times 7 \times 13$

3 MARKS

23. There is a circular path around a sports field. Sonia takes 18 minutes for one round of the field and Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time. After how many minutes will they meet again at the starting point? **(2020-B)**

Ans. $\text{LCM of } 12 \text{ and } 18 = 36$

\therefore They will meet again after 36 minutes

2	12, 18
3	6, 9
	2, 3

24. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? **(2020-B)**

Ans. $612 = 2^2 \times 3^2 \times 17$

$$48 = 2^4 \times 3$$

$$\text{HCF}(612, 48) = 2^2 \times 3 = 12$$

$$\text{Number of column} = 12$$

2	612	2	48
2	306	2	24
3	153	2	12
3	51	2	6
	17		3

25. Find HCF and LCM of 404 and 96 and verify that $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$. **(2018)**

Ans. $404 = 2 \times 2 \times 101 = 2^2 \times 101$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$$

$$\therefore \text{HCF of } 404 \text{ and } 96 = 2^2 = 4$$

$$\text{LCM of } 404 \text{ and } 96 = 101 \times 2^5 \times 3 = 9696$$

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

$$\text{Also } 404 \times 96 = 38784$$

$$\text{Hence } \text{HCF} \times \text{LCM} = \text{Product of } 404 \text{ and } 96$$

2	96	2	404
2	48	2	202
2	24		101
2	12		
2	6		
	3		

26. The HCF of 65 and 117 is expressible in the form $65m - 117$. Find the value of m . Also find the LCM of 65 and 117 using prime factorization method. **(2020 S, 2018, 13,12)**

Ans. $117 = 65 \times 1 + 52$

$$65 = 52 \times 1 + 13$$

$$52 = 13 \times 4 + 0$$

$$\therefore \text{HCF} = 13$$

$$\text{But } 65m - 117 = 13$$

$$65m = 117 + 13 = 130$$

$$\Rightarrow m = 2$$

$$\text{Now } 65 = 13 \times 5$$

$$\text{and } 117 = 3^2 \times 13$$

$$\therefore \text{LCM} = 13 \times 5 \times 3^2 = 585$$

Similar Problems for Practice

1. Find the HCF and LCM of 510 and 92 and verify that :

$$\text{HCF} \times \text{LCM} = \text{Product of two given numbers.}$$

(2015)

2. Find the HCF and LCM of the numbers 348 and 522 and also verify that $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$. **(2010, 2012)**

Ans. 1. 2, 23460

2. 174, 1044

27. If d is the HCF of 45 and 27, find x and y satisfying $d = 27x + 45y$.

(2011, 2013)

Real Numbers

Ans. $45 = 3 \times 3 \times 5 = 3^2 \times 5$
 $27 = 3 \times 3 \times 3 = 3^3$

$\therefore \text{HCF of } (45, 27) = 3^2 = 9$

We have, $d = 27x + 45y$
 $9 = 27 \times 2 + 45 \times -1$
 $\Rightarrow x = 2, y = -1$

28. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again ? **(2013)**

Ans. $48 = 2^4 \times 3$
 $72 = 2^3 \times 3^2$
 $108 = 2^2 \times 3^3$
 $\therefore \text{LCM} = 2^4 \times 3^3 = 432 \text{ seconds} = 7 \text{ min } 12 \text{ sec}$

i.e., they change at 07 : 07 : 12

29. Sita takes 35 seconds to pack and label a box. For Ram, the same job takes 42 seconds and for Geetha, it takes 28 seconds. If they all start using labelling machines at the same time, after how many seconds will they be using the labelling machines together. **(2015)**

Ans. We have to find the LCM of 35, 42 and 28

$$\begin{array}{r|l} 7 & 35, 42, 28 \\ \hline 2 & 5, 6, 4 \\ \hline & 5, 3, 2 \end{array}$$

$\therefore \text{LCM} = 7 \times 2 \times 5 \times 3 \times 2 = 420 \text{ sec}$

i.e., they will be using labelling machine together again after 420 seconds.

4 MARKS

30. Show that $(12)^n$ cannot end with digit 0 or 5 for any natural number n. **(2020 - S)**

Ans. If any number ends with the digit 0 or 5, it is always divisible by 5.

If 12^n ends with the digit zero it must be divisible by 5.

This is possible only if prime factorization of 12^n contains the prime number 5.

Now, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$\Rightarrow 12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

So the factorization does not contain 5.

Hence, there is no value of n (Natural number) for which 12^n ends with digit zero or five.

31. State Fundamental theorem of arithmetic.

Is it possible for the HCF and LCM of two numbers to be 18 and 378 respectively. Justify your answer. **(2015)**

Ans. Given, HCF = 18
 LCM = 378

$$\frac{\text{LCM}}{\text{HCF}} = \frac{378}{18} = 21$$

Thus HCF divides LCM exactly. So two numbers with HCF and LCM as 18 and 378 are possible.

32. What is the HCF and LCM of two prime numbers a and b?

Three alarm clocks ring at intervals of 6, 9 and 15 minutes respectively. If they start ringing together, after what time will they next ring together. **(2015)**

Ans. HCF = 1, LCM = a x b

Here we have to find the LCM of 6, 9 and 15.

$$3 \begin{array}{l} 6, 9, 15 \\ \hline 2, 3, 5 \end{array}$$

$$\text{LCM} = 3 \times 2 \times 3 \times 5 = 90$$

i.e., after 90 minutes they will start ringing together next time.

33. Jenny and Sally bought a special 360 day joint membership of a tennis club. Jenny will use the club every alternate day and Sally will use the club every third day. They both use the club on the first day. How many days will neither person use the club in the 360 days? **(2015)**

Ans. Jenny will come to tennis club for $\frac{360}{2} = 180$ days (\because She comes every 2nd day)

Sally will come to tennis club for days $\frac{360}{3} = 120$ days (\because she comes every 3rd day)

$$\text{Common days} = \frac{360}{6} = 60 \text{ days}$$

Total number of days any one of them comes to tennis club

$$= 180 + 120 - 60 = 240 \text{ days}$$

\Rightarrow Number of days neither person use the club

$$= \text{Total} - \text{any one or both comes to club} = 360 - 240 = 120 \text{ days}$$

EXERCISE 1.2

Topics : Irrational Numbers

3 MARKS

1. Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number. **(2020-S)**

Ans. Let us assume that $5 + 2\sqrt{7}$ is not an irrational number.

Real Numbers

$\therefore 5 + 2\sqrt{7}$ is a rational number p i.e. $5 + 2\sqrt{7} = p$

$$\Rightarrow \sqrt{7} = \frac{p-5}{2}$$

Which is a contradiction as RHS is a rational but LHS is irrational.

Hence $5 + 2\sqrt{7}$ can not be rational, so irrational.

2. If $\sqrt{2}$ is given as an irrational number, then prove that $(7 - 2\sqrt{2})$ is an irrational number.

Ans. Let $7 - 2\sqrt{2}$ be a rational number. **(2020 - B)**

$$\text{Then } 7 - 2\sqrt{2} = \frac{a}{b} \text{ where } a \text{ and } b \text{ are co-primes, } b \neq 0$$

$$\text{i.e., } 2\sqrt{2} = 7 - \frac{a}{b} = \frac{7b - a}{b}$$

$$\therefore \sqrt{2} = \frac{7b - a}{2b}$$

Since a and b are integers, $\frac{7b - a}{2b}$ is a rational number.

$\Rightarrow \sqrt{2}$ is a rational number. But we know that $\sqrt{2}$ is irrational which is a contradiction to our assumption and hence $7 - 2\sqrt{2}$ is irrational.

3. Prove that $\sqrt{2}$ is an irrational number. **(2020 B, 2019)**

Ans. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and $b \neq 0$.

$$\text{So } \sqrt{2} = \frac{a}{b}$$

$$\Rightarrow 2 = \frac{a^2}{b^2} \text{ (Squaring both sides)}$$

$$\Rightarrow a^2 = 2b^2$$

Thus a^2 is a multiple of 2

$\Rightarrow a$ is a multiple of 2.

Let $a = 2m$ for some integer m

$$\therefore b^2 = 2m^2$$

Thus b^2 is a multiple of 2

$\Rightarrow b$ is a multiple of 2

Hence 2 is a common factor of a and b .

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number.

4. Prove that $\sqrt{3}$ is an irrational number. **(2020 B, 2019)**

Ans. Let us assume that $\sqrt{3}$ be a rational number

$$\therefore \sqrt{3} = \frac{p}{q} \text{ where } p \text{ and } q \text{ are co-primes and } q \neq 0$$

$$\Rightarrow p^2 = 3q^2 \dots\dots\dots(1)$$

$$\therefore 3 \text{ divides } p^2$$

$$\text{i.e., } 3 \text{ divides } p \text{ also } \dots\dots\dots(2)$$

$$\text{Let } p = 3m, \text{ for some integer } m$$

$$\text{From (1), } 9m^2 = 3q^2$$

$$\Rightarrow q^2 = 3m^2 \quad \therefore 3 \text{ divides } q^2$$

$$\text{i.e., } 3 \text{ divides } q \text{ also } \dots\dots\dots(3)$$

From (2) and (3), we get that 3 divides p and q both which is a contradiction to the fact that p and q are co-primes.

Hence our assumption is wrong.

$\therefore \sqrt{3}$ is irrational

5. Show that $\frac{3 + \sqrt{7}}{2}$ is an irrational number, given that $\sqrt{7}$ is irrational. **(2019)**

Ans. Let $\frac{3 + \sqrt{7}}{2}$ be a rational number.

$$\Rightarrow \frac{3 + \sqrt{7}}{2} = \frac{p}{q}, \quad q \neq 0$$

$$\Rightarrow 3q + \sqrt{7}q = 2p$$

$$\Rightarrow \sqrt{7}q = 2p - 3q$$

$$\Rightarrow \sqrt{7} = \frac{2p - 3q}{q}$$

RHS is a rational no. whereas LHS is an irrational number which is wrong.

$\therefore \frac{3 + \sqrt{7}}{2}$ is an irrational number.

6. Prove that $\sqrt{5}$ is irrational. **(2020 S, 2019, 2012, 2011, 2010)**

Ans. Let us assume to the contrary that $\sqrt{5}$ is rational.

$$\text{i.e., } \sqrt{5} = \frac{p}{q}, \text{ where both } p \text{ and } q \text{ are co-primes}$$

$$\text{i.e., } p = \sqrt{5} q$$

On squaring both sides, $p^2 = 5q^2 \rightarrow \textcircled{1}$

So p^2 is divisible by 5. So p is also divisible by 5.

$$\text{Put } p = 5C$$

Real Numbers

$$\Rightarrow p^2 = 25 C^2$$

$$\text{from } \textcircled{1} 5q^2 = 25 C^2 \Rightarrow q^2 = 5C^2$$

$\therefore q^2$ is divisible by 5. So q is also divisible by 5. This contradicts the facts that p and q are coprimes. So our assumption is wrong and $\sqrt{5}$ is irrational.

7. Show that $(\sqrt{3} + \sqrt{5})^2$ is an irrational number. **(2015)**

Ans. Let if possible $(\sqrt{3} + \sqrt{5})^2$ be a rational number.

$$\therefore (\sqrt{3} + \sqrt{5})^2 = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are co-primes}$$

$$\text{ie. } 3 + 5 + 2\sqrt{15} = \frac{p}{q}$$

$$2\sqrt{15} = \frac{p}{q} - 8 = \frac{p - 8q}{q}$$

$$\sqrt{15} = \frac{p - 8q}{2q}$$

Since p and q are integers, $\frac{p - 8q}{2q}$ is rational and $\sqrt{15}$ is irrational. But we know that $\sqrt{15}$ is irrational

\therefore Our assumption is wrong and $(\sqrt{3} + \sqrt{5})^2$ is an irrational number.

8. If x is rational and \sqrt{y} is irrational, then prove that $x + \sqrt{y}$ is irrational. **(2010, 2011)**

Ans. Let us assume to the contrary that $x + \sqrt{y}$ is rational.

$$\therefore x + \sqrt{y} = \frac{p}{q}, p \text{ and } q \text{ are coprimes.}$$

$$\text{We have } \sqrt{y} = \frac{p}{q} - x = \frac{p - qx}{q}$$

Since p and q are integers $\frac{p - qx}{q}$ is rational

But \sqrt{y} is irrational. [Given]

$$\therefore \frac{p - qx}{q} \text{ is irrational} \Rightarrow x + \sqrt{y} \text{ is irrational.}$$

4 MARKS

9. Prove that $(\sqrt{2} + \sqrt{5})$ is irrational. **(2020 - S)**

Ans. Let us assume that $\sqrt{5} + \sqrt{2}$ is rational.

$$\text{i.e., } \sqrt{5} + \sqrt{2} = \frac{p}{q}, q \neq 0 \text{ where } p \text{ and } q \text{ are coprimes.}$$

On squaring both sides, we have $(\sqrt{5} + \sqrt{2})^2 = \left(\frac{p}{q}\right)^2$

$$5 + 2 + 2\sqrt{5} \times \sqrt{2} = \frac{p^2}{q^2}$$

$$7 + 2\sqrt{10} = \frac{p^2}{q^2}$$

$$2\sqrt{10} = \frac{p^2}{q^2} - 7 = \frac{p^2 - 7q^2}{q^2}$$

$$\sqrt{10} = \frac{p^2 - 7q^2}{2q^2}$$

Since p and q are integers, $\frac{p^2 - 7q^2}{2q^2}$ is rational $\Rightarrow \sqrt{10}$ is rational.

But by the method of contradiction, we have $\sqrt{10}$ is irrational.

So our assumption is wrong and $\sqrt{5} + \sqrt{2}$ is irrational

10. Prove that $1 + \frac{1}{\sqrt{2}}$ is irrational.

(2015)

Ans. Let us assume to the contrary that $1 + \frac{1}{\sqrt{2}}$ is rational.

$$\text{i.e., } 1 + \frac{1}{\sqrt{2}} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are coprimes.}$$

$$\text{i.e., } \frac{\sqrt{2} + 1}{\sqrt{2}} = \frac{p}{q}$$

$$\text{i.e., } \frac{(\sqrt{2} + 1) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{p}{q}$$

$$\text{i.e., } \frac{2 + \sqrt{2}}{2} = \frac{p}{q}$$

$$2 + \sqrt{2} = \frac{2p}{q}$$

$$\sqrt{2} = \frac{2p}{q} - 2 = \frac{2p - 2q}{q} = \frac{2(p - q)}{q}$$

Since p and q are integers, $\frac{2(p - q)}{q}$ is rational $\Rightarrow \sqrt{2}$ is rational.

But we know that $\sqrt{2}$ is irrational by the method of contradiction. So our assumption is wrong and $1 + \frac{1}{\sqrt{2}}$ is irrational.

Similar Problems for Practice

1. Show that $5 - 2\sqrt{3}$ is an irrational number. (2015)

2. Prove that $2\sqrt{2}$ is an irrational number. (2015)

ASSERTION & REASONING

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct

Real Numbers

explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. Assertion (A) : If HCF of 510 and 92 is 2, then the LCM of 510 & 92 is 32460
Reason (R) : as $HCF(a,b) \times LCM(a,b) = a \times b$
2. Statement A (Assertion): If product of two numbers is 5780 and their HCF is 17, then their LCM is 340
Statement R(Reason) : HCF is always a factor of LCM
3. Assertion : HCF of 26 and 91 is 13
Reason : the prime factorization of $26 = 2 \times 13$ and $91 = 7 \times 13$
4. Assertion: the multiplication of two irrational no. is may be rational or irrational
Reason: the product of two irrational no.is always rational
5. Assertion: the largest number that divide 70 & 125 which leaves remainder 5 & 8 is 13
Reason : $HCF(65,117) = 13$

Ans. 1. (d) 2. (b) 3. (a) 4. (c) 5. (a)

1 MARK

1. $\left(\frac{2 + \sqrt{5}}{3}\right)$ is _____ number. **(2020-S)**

Ans. irrational

2. Given that $HCF(135, 225) = 45$, find the LCM (135, 225). **(2020-S)**

Ans. $LCM = \frac{135 \times 225}{45} = 675$

3. Write one irrational number between 0.15 and 0.21. **(2020-S)**

Ans. 0.15010010001 ...

4. Find the HCF of 12, 18 and 30. **(2020)**

Ans. $HCF = 2 \times 3 = 6$

$$\begin{array}{r|l} 2 & 12, 18, 30 \\ 3 & 6, 9, 15 \\ \hline & 2 \quad 3 \quad 5 \end{array}$$

5. After how many decimal places will the decimal representation of the rational number $\frac{229}{2^2 \times 5^7}$ terminate? **(2020-S)**

Ans. After 7 decimal places

6. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other. **(2020-S)**

Ans. $b = \frac{HCF \times LCM}{a} = \frac{182 \times 13}{26} = 91$

7. If $\text{HCF}(336, 54) = 6$, find $\text{LCM}(336, 54)$. (2019)

$$\text{Ans. } \text{LCM}(336, 54) = \frac{a \times b}{\text{HCF}} = \frac{336 \times 54}{6} = 3024$$

8. The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab .

$$\text{Ans. } a \cdot b = \text{LCM} \times \text{HCF} = 1000 \quad (2019)$$

9. Find after how many places of decimal the decimal form of the number $\frac{27}{2^3 \cdot 5^4 \cdot 3^2}$ will terminate. (2019)

$$\text{Ans. } \frac{27}{2^3 \cdot 5^4 \cdot 3^2} = \frac{3}{2^3 \cdot 5^4}$$

It will terminate after 4 decimal places

10. Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$. (2019)

$$\text{Ans. } 2^2 \times 5^2 \times 5 \times 3^2 \times 17 = (10)^2 \times 5 \times 3^2 \times 17$$

\therefore No. of zeroes in the end of the number = Two

11. The decimal expansion of $\frac{51}{2^3 \times 5^2}$ will terminate after how many decimal places ?

Ans. 3 decimal places

12. Express 429 as a product of its prime factors. (2019)

$$\text{Ans. } 429 = 3 \times 11 \times 13$$

13. Find a rational number between $\sqrt{2}$ and $\sqrt{7}$. (2019)

$$\text{Ans. } \sqrt{2} = 1.41, \sqrt{7} = 2.6$$

\therefore Rational number lying between $\sqrt{2}$ and $\sqrt{7}$ is 1.43 [Or any other answer]

14. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$. (2019)

Ans. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.)
e.g., 1.5, 1.6, 1.63 etc.

15. The decimal expansion of $\frac{135}{2^25^43^2}$ will terminate after how many places of decimals?

$$\text{Ans. } \frac{135}{2^25^43^2} = \frac{15}{2^25^4} = \frac{3}{2^25^3} \quad (2019)$$

It will terminate after 3 places of decimal.

16. Write whether the rational number $\frac{13}{3125}$ has a decimal expansion which is terminating or non-terminating repeating. (2018)

$$\text{Ans. } 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5 \times 2^0$$

$\therefore \frac{13}{3125}$ is terminating decimal expansion.

Multiple Choice Questions :

1. The sum of exponents of prime factors in the prime-factorisation of 196 is **(2020-S)**
 (a) 3 (b) 4 (c) 5 (d) 2
2. The decimal representation of $\frac{117}{2^3 5^4 3^2}$ will **(2020-S)**
 (a) terminate after 3 decimal places (b) terminate after 2 decimal places
 (c) terminate after 4 decimal places (d) not terminate
3. Euclid's division Lemma states that for two positive integers a and b, there exists unique integer q and r satisfying $a = bq + r$, and **(2020-S)**
 (a) $0 < r < b$ (b) $0 < r \leq b$ (c) $0 \leq r < b$ (d) $0 \leq r \leq b$
4. The total number of factors of a prime number is **(2020-S)**
 (a) 1 (b) 0 (c) 2 (d) 3
5. The HCF and the LCM of 12, 21, 15 respectively are **(2020-S)**
 (a) 3, 140 (b) 12, 420 (c) 3, 420 (d) 420, 3 **(2020-S)**
6. The LCM of the smallest two-digit number and the largest multiple of 6 which is less than 50 is **(2020-S)**
 (a) 2 (b) 48 (c) 120 (d) 240
7. HCF of two numbers is 27 and their LCM is 162. If one of the number is 54, then the other number is **(2020-B)**
 (a) 36 (b) 35 (c) 9 (d) 81
8. HCF of 144 and 198 is **(2020-B)**
 (a) 9 (b) 18 (c) 6 (d) 12
9. Given that $\text{HCF}(156, 78) = 78$, $\text{LCM}(156, 78)$ is **(2020-B)**
 (a) 156 (b) 78 (c) 156×78 (d) 156×2
10. $2\sqrt{3}$ is **(2020-B)**
 (a) an integer (b) a rational number
 (c) an irrational number (d) a whole number
11. 225 can be expressed as **(2020-B)**
 (a) 5×3^2 (b) $5^2 \times 3$ (c) $5^2 \times 3^2$ (d) $5^3 \times 3$
12. 120 can be expressed as a product of its prime factors as **(2020-B)**
 (a) $5 \times 8 \times 3$ (b) 15×2^3 (c) $10 \times 2^2 \times 3$ (d) $5 \times 2^3 \times 3$
13. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal? **(2020-B)**
 (a) 2 (b) 4 (c) 5 (d) 1
14. $2.\overline{35}$ is

- (a) an integer (b) a rational number (2020-B)
 (c) an irrational number (d) a natural number
15. 840 can be expressed as a product of prime numbers as: (2020-B)
 (a) $2^2 \times 6 \times 5 \times 7$ (b) $2^3 \times 3 \times 5 \times 7$
 (c) $2 \times 3 \times 4 \times 5 \times 7$ (d) $3 \times 5 \times 7 \times 8$
16. Which of the following is the decimal expansion of an irrational number? (2020-B)
 (a) 3.14 (b) 3.333... (c) 6.010010001... (d) 7.25
- Ans.** 1. (B) 2. (C) 3. (C) 4. (C) 5. (C) 6. (D) 7. (D)
 8. (B) 9. (A) 10. (C) 11. (C) 12. (D) 13. (C) 14. (B)
 15. (B) 16. (C)

SELF - ASSESSMENT TEST

1. The HCF of 135 and 225 is: (2020 - B)
 (a) 9 (b) 15 (c) 45 (d) 25
2. The exponent of 2 in the prime factorization of 144, is (2020 - S)
 (a) 2 (b) 4 (c) 1 (d) 6
3. The HCF of 135 and 225 is (2020 - S)
 (a) 15 (b) 75 (c) 45 (d) 5
4. The simplest form of $\frac{1095}{1168}$ is (2020 - B)
 (a) $\frac{17}{26}$ (b) $\frac{25}{26}$ (c) $\frac{13}{16}$ (d) $\frac{15}{16}$
5. Which of the following rational numbers is expressible as a terminating decimal?
 (a) $\frac{124}{165}$ (b) $\frac{131}{30}$ (c) $\frac{2027}{625}$ (d) $\frac{1625}{462}$ (2020 - B)
6. Write whether $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ on simplification gives an irrational or a rational number. (2018)
7. Find HCF of 144, 96 and 54 by prime factorisation method. (2015)
8. State fundamental theorem of Arithmetic. Using it check whether there is any value of n for which 5^n ends with the digit two. For solutions (2015)
9. Show that $3\sqrt{7}$ is an irrational number. (2015)
10. Prove that $\sqrt{6} + \sqrt{2}$ is irrational. (2011)

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