TEACHERS FORUM®

QUESTION BANK
(solved)

Based on CBSE Previous years' question papers

Class XII

PHYSICS

SUBJECT EXPERTS

CONTENTS

1 ELECTRIC CHARGES AND FIELDS

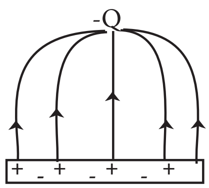
1 MARKS

1. If the electric flux entering and leaving a closed surface in air are ϕ_1 and ϕ_2 respectively, the net electric charge enclosed within the surface is _______. **(2020)**

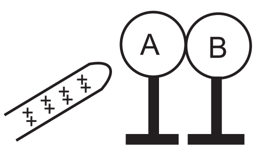
Ans. Net charge = $(\phi_1 - \phi_2)\epsilon_0$

2. Draw the pattern of electric field lines, when a point charge – Q is kept near an uncharged conducting plate. **(2019)**

Ans.

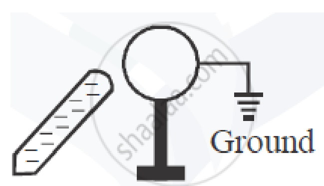


3. Two metallic spheres A and B kept on insulating stands are in contact with each other. A positively charged rod P is brought near the sphere A as shown in the figure. The two spheres are separated from each other, and the rod P is removed. What will be the nature of charges on spheres A and B ? **(2019)**



Ans. Sphere A will be negatively charged. Sphere B will be positively charged.

4. A metal sphere is kept on an insulating stand. A negatively charged rod is brought near it, then the sphere is earthed as shown. On removing the earthing and taking the negatively charged rod away, what will be the nature of charge on the sphere ? Give reason for your answer. **(2019)**



Ans. Sphere will be positively charged

Reason - Electrostatic Induction.

5. Two identical conducting balls A and B have charges –Q and +3Q respectively. They are brought in contact with each other and then separated by a distance d apart. Find the nature of the Coulomb force between them. **(2019)**

Ans. Repulsive

6. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the shell. What will be the surface charge density on the (i) inner surface, and (ii) outer surface of the shell ? **(2019)**

Ans. Surface charge density on inner surface = $-\frac{Q}{4\pi R_1^2}$

Surface charge density on Outer surface = $+\frac{Q}{4\pi R_2^2}$

7. Under what condition will the current in a wire be the same when connected in series and in parallel of n identical cells each having internal resistance r and external resistance R ? **(2019)**

Ans. When $R + nr = nR + r$

8. Why is the electrostatic potential inside a charged conducting shell constant throughout the volume of the conductor ? **(2019)**

Ans. E = 0 inside the conductor & has no tangential component on the surface.

$\therefore$ No work is done in moving charge inside or on the surface of the conductor & Potential is constant.

9. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor ? **(2019)**

Ans. On equipotential surface $\Delta V = 0$

As $\Delta V = -\vec{E}\,\Delta\vec{r}$

$\therefore$ $\vec{E}$ is perpendicular to $\Delta\vec{r}$

10. Why are manganin and constantan widely used for making wire bound standard resistors ? **(2019)**

Ans. Manganin and constantan are used for making wire bound standard resistors because of small temperature co-efficient and high resistivity.

11. Define Electric Flux. Write its SI unit. **(2016)**

Ans. Electric flux $\Delta\Phi$, through an area element $\overrightarrow{\Delta S}$, is defined by

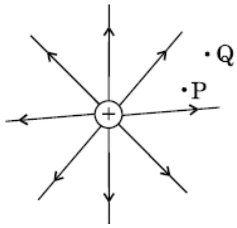
$\Delta\Phi = \vec{E}.\overrightarrow{\Delta S} = E\Delta S \cos\theta$, where θ is the angle between $\vec{E}$ and $\overrightarrow{\Delta S}$.

S.I unit of electric flux is $NC^{-1}m^2$.

12. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased ? **(2016)**

Ans. Electric flux remains unaffected.

13. The figure shows the field lines of a positive point charge. What will be the sign of the potential energy difference of a small negative charge between the points Q and P ? Justify your answer. **(2015)**



Ans. Positive.

Reason: Negative charge moves from a point at a lower potential energy to one at a higher potential energy.

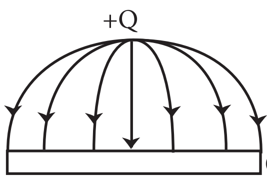
14. Define dielectric constant of a medium. What is its S.I. unit ? **(2015)**

Ans. Dielectric Constant of a medium is the ratio of intensity of electric field in free space to that in the dielectric medium.

S.I. Unit : No Unit

15. A point charge +Q is placed in the vicinity of a conducting surface. Trace the field lines between the charge and the conducting surface. **(2015)**

Ans.

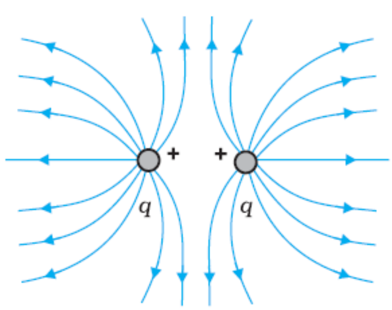


16. Why do the electrostatic field lines not form closed loops ? **(2015)**

Ans. Due to conservative nature of electric field.

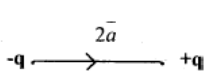
17. Depict the electric field lines due to two positive charges kept a certain distance apart.

Ans. **(2015)**



18. Define electric dipole moment. Write its S.I. unit. **(2011)**

Ans. Electric dipole moment is defined as the numerical product of charge and distance between the charges, and is directed from negative to positive charge.



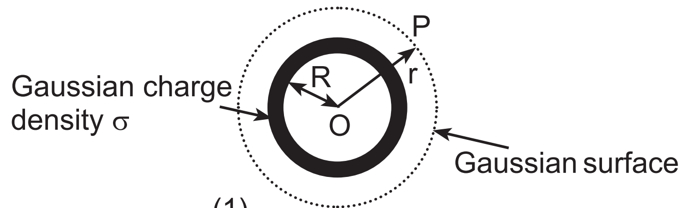
$\vec{p} = q(2\vec{a})$

Unit : coulomb metre

2 MARKS

19. Apply Gauss's law to show that for a charged spherical shell, the electric field outside the shell is, as if the entire charge were concentrated at the centre. **(2019)**

Ans. Flux through the small section of Gaussian surface



$\phi = \oint \vec{E} \cdot \vec{ds}$

$\therefore \quad \phi = \oint E ds \cos\theta$

$\because E \parallel \vec{ds}, \ \theta = 0$

$\therefore \quad \phi = E, 4\pi R^2$ .... (1)

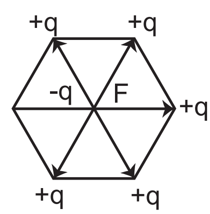
Applying Gauss's theorem

$\phi = \frac{q}{\varepsilon_0}$ .... (2)

from equations 1 and 2, $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$

20. Five point charges, each of charge + q are placed on five vertices of a regular hexagon of side 'l'. Find the magnitude of the resultant force on a charge – q placed at the centre of the hexagon. **(2019)**

Ans. The forces due to the charges placed diagonally opposite at the vertices of hexagon, on the charge -q cancel in pairs. Hence net force is due to one charge only.



Net Force $|\vec{F}| = \frac{1}{4\pi\varepsilon_0} \frac{q}{l^2}$.

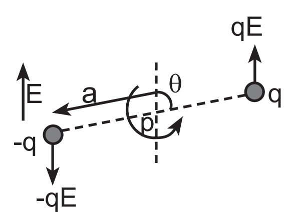
21. Derive an expression for the torque acting on an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Write the direction along which the torque acts.

Ans. Force on either charge F = qE **(2019)**

Magnitude of torque = Either of force × ⊥ distance between them.

$\tau = qE\, 2a \sin\theta$

$\tau = pE \sin\theta$

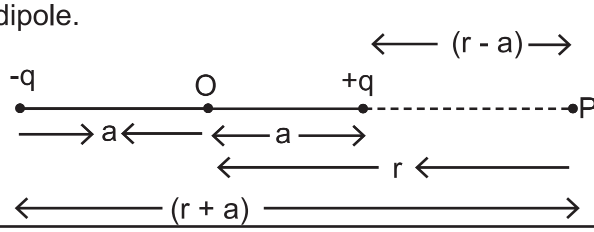


$\vec{r} = \vec{p} \times \vec{E}$

Direction is normal to the paper coming out of it.

22. Derive an expression for the electric field at a point on the axis of an electric dipole of dipole moment $\vec{p}$. Also write its expression when the distance r >> the length 'a' of the dipole. **(2019)**

Ans.



$$E_- = \frac{q}{4\pi\varepsilon_0 (r+a)^2} \text{ along } (-)\vec{p}$$

$$E_+ = \frac{q}{4\pi\varepsilon_0 (r+a)^2} \text{ along } \vec{p}$$

Total field at P, $E = E_- - E_+$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

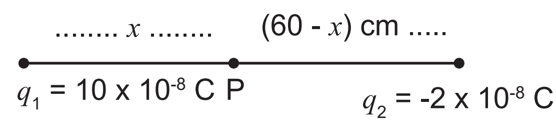
$$= \frac{q}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}$$

For r >> a

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

23. Two point charges, $q_1 = 10 \times 10^{-8}$ C and $q_2 = -2 \times 10^{-8}$ C are separated by a distance of 60 cm in air. Find at what distance from the charge q_1, would the electric potential be zero? **(2018)**

Ans.



We have $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$$\frac{1}{4\pi\epsilon_0}\left[\frac{10 \times 10^{-8}}{x \times 10^{-2}} + \frac{-2 \times 10^{-8}}{(60-x)\times 10^{-2}}\right] = 0$$

$$\Rightarrow \frac{5}{x} = \frac{1}{(60-x)}$$

$$\Rightarrow 6x = 300$$

$$\Rightarrow x = 50 \text{ cm (from } q_1)$$

24. Derive an expression for the work done in rotating a dipole from the angle θ_0 to θ_1 in a uniform electric field E. **(2016)**

Ans. Work done against the restoring torque, $dw = \tau\, d\theta$

$$dw = pE \sin\theta\, d\theta$$

$$\therefore \quad W = pE\int_{\theta_0}^{\theta_1} \sin\theta\, d\theta$$

$$= pE \cos\theta_0 - \cos\theta_1$$

25. Define the term 'electric flux'. Write its SI units. What is the flux due to electric field $\vec{E} = 3 \times 10^3 \hat{i}$ N/C through a square of side 10 cm, when it is held normal to $\vec{E}$? **(2015)**

Ans. Electric flux is the number of electric field lines passing through a given area.

$$d\Phi = \vec{E}.d\vec{s}$$

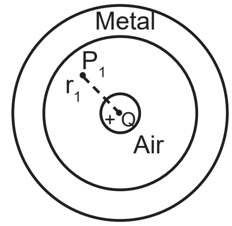
OR $\Phi = \int_s \vec{E}.d\vec{s}$

SI units : $\left(\frac{N - m^2}{C}\right)$ or (V - m)

$$\Phi = \vec{E} \cdot \vec{S} = ES \text{ (as } \theta = 0^o)$$

$$= 3 \times 10^3 \times (10 \times 10^{-2})^2 = 30 \frac{N - m^2}{C}$$

26. A small metal sphere carrying charge +Q is located at the centre of a spherical cavity in a large uncharged metallic spherical shell. Write the charges on the inner and outer surfaces of the shell. Write the expression for the electric field at the point P_1. **(2014)**



Ans. Charge on inner surface : - Q

Charge on outer surface : + Q

Electric field at point P_1, $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R_1^2}$

27. An electric dipole is held in a uniform electric field.

(i) Show that the net force acting on it is zero.

(ii) The dipole is aligned parallel to the field. Find the work done in rotating it through the angle of 180°. **(2012)**

Ans. (i) $\vec{F_1} = q\vec{E}$ and $\vec{F_2} = -q\vec{E}$

$$\vec{F}_{net} = \vec{F_1} + \vec{F_2}$$

$$\therefore F_{net} = 0$$

(ii) $W = EP(\cos\theta_1 - \cos\theta_2)$

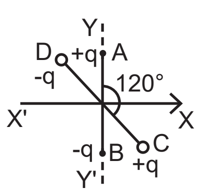
$$W = 2EP$$

28. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. **(2011)**

Ans. Charge enclosed by the cylindrical surface $q = \lambda l$

flux $\varphi = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

29. Two small identical electrical dipoles AB and CD, each of dipole moment 'p' are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination ? If this system is subjected to electric field ($\vec{E}$) directed along + X direction, what will be the magnitude and direction of the torque acting on this ? **(2011)**



Ans. Resultant dipole moment = p

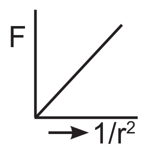
The magnitude of torque is pE sin 30° = $\frac{pE}{2}$

The direction of torque is clockwise when viewed from above

(or $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$).

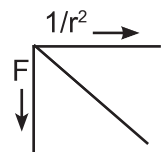
30. Plot a graph showing the variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$; where r is the distance between the two charges of each pair of charges: (1 μC, 2 μC) and (2 μC, – 3 μC). Interpret the graphs obtained. **(2011)**

Ans. (a)



Repulsive

(b)



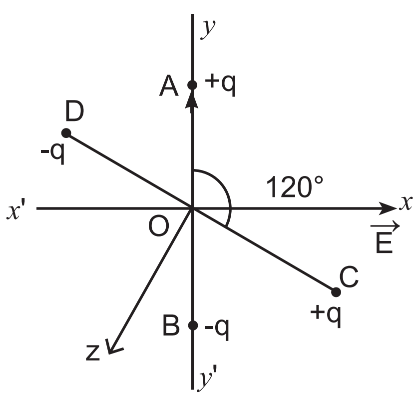
Attractive

3 MARKS

31. Two small identical electric dipoles AB and CD, each of dipole moment $\vec{p}$ are kept at an angle of 120° to each other in an external electric field $\vec{E}$ pointing along the x-axis as shown in the figure. Find the

(a) Dipole moment of the arrangement, and

(b) Magnitude and direction of the net torque acting on it. **(2020)**



Ans. (a) Given $p_A = p_c = p$

Resultant dipole moment,

$$P_r = \sqrt{p^2 + p^2 + 2 \times p^2 \times \cos 120°}$$

$$= \sqrt{2p^2 + 2p^2\left(-\frac{1}{2}\right)} = p$$

This dipole moment acts along the bisector of

∠AOC i.e. at an angle of 30° with + X direction.

(b) Torque, $\tau = pE \sin 30° = \frac{1}{2} pE$

By right hand rule, the torque τ acts into the plane of paper along Z-direction.

32. Using Gauss' law, derive an expression for the electric field at a point near an infinitely long straight uniformly charged wire. **(2019)**

Ans. Flux through the Gaussian Surface = Flux through the curved Cylindrical part

$= E \times 2\pi r l$

The surface includes charge equal to λl

Gauss's Law then gives $E.2\pi r l = \frac{\lambda l}{\epsilon_\circ}$

$$E = \frac{\lambda}{2\pi\epsilon_\circ r}$$

33. (a) An electric dipole of dipole moment $\vec{p}$ is held in a uniform electric field $\vec{E}$. Show that the torque acting on the dipole is given by $\vec{p} \times \vec{E}$. **(2019)**

(b) How much work is required in turning the electric dipole from the position of most stable equilibrium to that of most unstable ?

Ans. (a) Magnitude of torque $= qE \times 2a \sin\theta$

$= 2qaE \sin\theta$

$= pE \sin\theta$

Its direction is normal to the plane of the paper, coming out of it.

The magnitue of $\vec{P} \times \vec{E}$ Is also $PE \sin\theta$ and its direction is normal to the paper,

Thus $\vec{\tau} = \vec{P} \times \vec{E}$

(b) Work done in turning the dipole from most stable euilibirium to most unstable equilibrium position

$$W = u_f - u_i$$

$$u = -pE \cos\theta$$

$$u_i = -pE \cos 0 = -pE$$

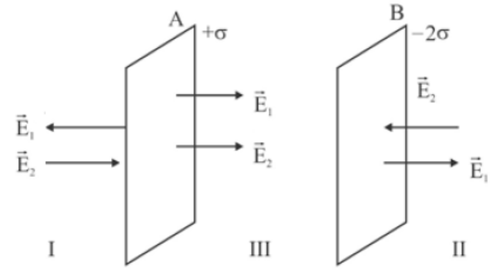
$$u_f = -pE \cos 180 = pE$$

$$W = pE - (-pE) = 2\,pE$$

34. Two large charged plane sheets of charge densities σ and -2σ C/m^2 are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points **(2019)**

(i) to the left of the first sheet,

(ii) to the right of the second sheet, and (iii) between the two sheets.



Ans. Electric field in the region left of first sheet,

$$E_I = E_1 + E_2$$

$$E_I = \frac{\sigma}{\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

$$E_I = +\frac{\sigma}{2\varepsilon_0}$$ It is towards right

Electric field in the region to the right of second sheet

$$E_{II} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{\varepsilon_0}$$

$$E_{II} = -\frac{\sigma}{2\varepsilon_0}$$ It is towards left

Electric field between the two sheets

$$E_{III} = E_1 + E_2$$

$$E_{III} = \frac{\sigma}{\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

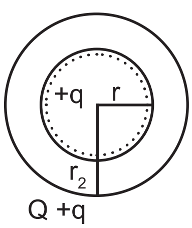
$$E_{III} = +\frac{3\sigma}{2\varepsilon_0}$$ Electric field is towards the right

35. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell. **(2019)**

(b) Is the electric field inside a cavity (with no charge) zero; independent of the fact whether the shell is spherical or not ? Explain.

Ans. (a)



The surface charge density on inner surface of the shell is $\sigma_1 = \frac{-q}{4\pi r_1^2}$

The surface charge density on outer shell is $\sigma_2 = \frac{Q+q}{4\pi r_2^2}$

(b) Consider a Gaussian surface inside the shell, net flux is zero since $q_{net} = 0$.

According to Gauss's law it is independent of shape and size of shell.

36. A charge Q is distributed over the surfaces of two concentric hollow spheres of radii r and R (R >> r), such that their surface charge densities are equal. Derive the expression for the potential at the common centre. **(2019)**

Ans. $Q = q_1 + q_2 = 4\pi\sigma (r^2 + R^2)$

Potential at common centre

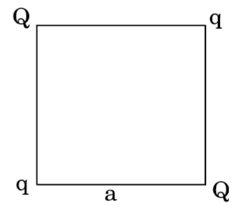
$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \times \left[\frac{4\pi r^2\sigma}{r} + \frac{4\pi R^2\sigma}{R}\right]$$

$$= \frac{(r + R)\sigma}{\varepsilon_0}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \left[\frac{Q\,(r + R)}{r^2 + R^2}\right]$$

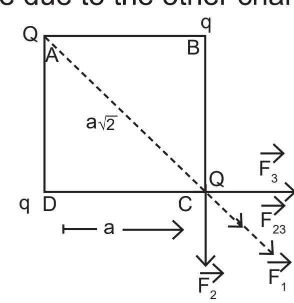
37. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure. **(2018)**



Find the (a) resultant electric force on a charge Q, and

(b) potential energy of this system.

Ans. (a) Let us find the force on the charge Q at the point C Force due to the other charge Q



$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{(a\sqrt{2})^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q^2}{2a^2}\right) \text{ (along AC)}$$

Force due to the charge q (at B), $F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{a^2}$ along BC

Force due to the charge q (at D), $F_3 = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{a^2}$ along DC

Resultant of these two equal forces, $F_{23} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ(\sqrt{2})}{a^2}$ (along AC)

$\therefore$ Net force on charge Q (at point C)

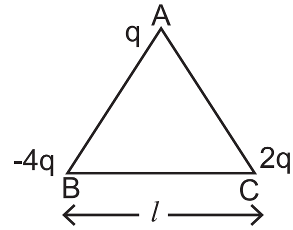
$$F = F_1 + F_{23} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a^2}\left[\frac{Q}{2} + \sqrt{2}q\right]$$

This force is directed along AC

(b) Potential energy of the system $= \dfrac{1}{4\pi\epsilon_0}\left[4\dfrac{qQ}{a} + \dfrac{q^2}{a\sqrt{2}} + \dfrac{Q^2}{a\sqrt{2}}\right]$

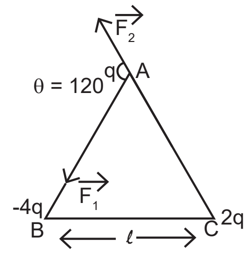
$$= \frac{1}{4\pi\epsilon_0 a}\left[4qQ + \frac{q^2}{\sqrt{2}} + \frac{Q^2}{\sqrt{2}}\right]$$

38. (a) Three point charges q, – 4q and 2q are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q. **(2018)**



(b) Find out the amount of the work done to separate the charges at infinite distance.

Ans. (a) Force on charge q due to the charge - $4q$,



$$F_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{4q^2}{l^2}\right), \text{ along AB}$$

Force on the charge q, due to the charge $2q$,

$$F_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{2q^2}{l^2}\right), \text{ along CA}$$

The forces F_1 and F_2 are inclined to each other at an angle of 120°

Hence, resultant electric force on charge q

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$
$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$
$$= \sqrt{F_1^2 + F_2^2 - F_1F_2}$$
$$= \left(\frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\right)\sqrt{16+4-8} = \frac{1}{4\pi\epsilon_0}\left(\frac{2\sqrt{3}q^2}{l^2}\right)$$

(b) Net P.E. of the system, $= \frac{1}{4\pi\epsilon_0}\frac{q^2}{l}\left[-4+2-8\right] = \frac{(-10)}{4\pi\epsilon_0}\frac{q^2}{l}$

$\therefore$ Work done $= \frac{10\,q^2}{4\pi\epsilon_0 l} = \frac{5\,q^2}{2\pi\epsilon_0 l}$

39. Define electric flux. Is it a scalar or a vector quantity? **(2018)**

A point charge causes an electric flux of $-1 \times 10^3 \frac{\text{Nm}^2}{\text{C}}$ to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. What is the value of the point charge?

Does the flux passing through the surface depend on the radius of the Gaussian surface enclosing the charge ? Justify your answer.

Ans. • Electric flux: It is defined as the number of electric filed lines passing through a surface placed perpendicular to the direction of electric field lines

• It is a Scalar Quantity

• $\Phi = -1 \times 10^3 \frac{Nm^2}{C}$

$$\Phi = \frac{q_{Enclosed}}{\epsilon_0}$$

$$q_{Enclosed} = \Phi\epsilon_0$$

$$= -1 \times 10^3 \times 8.854 \times 10^{-12}\text{ C} = -8.854 \times 10^{-9}\text{ C}$$

Or $q_{Enclosed} = -8.854$ nC

• No, it does not depend the on radius of the Gaussian surface.

Justification : As According to Gauss Theorem $\Phi = \frac{q_{Enclosed}}{\epsilon_0}$

Hence flux does not depend on radius.

40. A point charge causes an electric flux of $-4\pi \times 10^3$ Nm²/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (i) Calculate the value of the point charge. (ii) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface ? Justify your answer. **(2017)**

Ans. (i) From Gauss's Law $\varphi_E = \frac{q}{\varepsilon_0}$

$$\therefore q = \varepsilon_0 \varphi_E = 8.85 \times 10^{-12} \times (-4\pi \times 10^3)$$

$$= 0.111 \times 10^{-6}\ C = -0.111\ \mu C$$

(ii) Flux would remain same .i.e. $-4\pi \times 10^3\ Nm^2/c$, as flux depends on charge enclosed, not on the dimensions of Gaussian surface.

41. A long charged cylinder of linear charge density $+\lambda_1$ is surrounded by a hollow coaxial conducting cylinder of linear charge density $-\lambda_2$. Use Gauss's law to obtain expressions for the electric field at a point (i) in the space between the cylinders, and (ii) outside the larger cylinder. **(2017)**

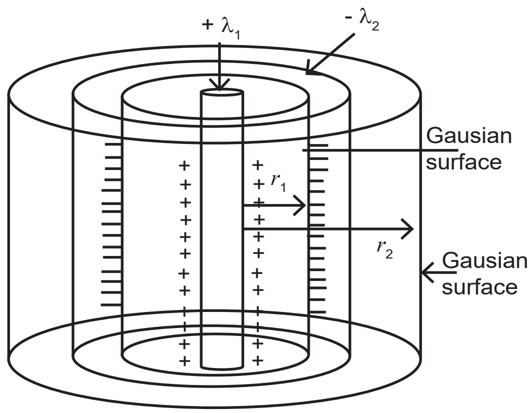
Ans. As Gauss's Law states, $\oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$

(i) $\oint \vec{E_1} \cdot \vec{ds} = \frac{\lambda_1 l}{\varepsilon_0}$

$\Rightarrow \vec{E_1} = \frac{\lambda_1}{2\pi\varepsilon_0 r_1} \hat{r}_1$

(ii) $\oint \vec{E_2} \cdot \vec{ds} = \frac{(\lambda_1 - \lambda_2) l}{\varepsilon_0}$

$\Rightarrow \vec{E_2} = \frac{(\lambda_1 - \lambda_2)}{2\pi\varepsilon_0 r_2} \hat{r}_2$

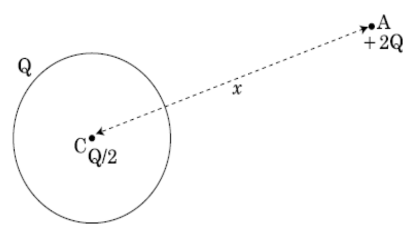


42. A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $Q/2$ is placed at the centre C and another charge $+2Q$ is placed outside the shell at A at a distance x from the centre as shown in the figure. **(2016)**

(i) Find the electric flux through the shell.

(ii) State the law used.

(iii) Find the force on the charges at the centre C of the shell and at the point A.



Ans. (i) Electric flux through a Gaussian surface, $\varphi = \frac{\text{total enclosed charge}}{\epsilon_0}$

Net charge enclosed inside the shell, $q = 0$

$\therefore$ Electric flux through the shell $\frac{q}{\epsilon_0} = 0$

(ii) Gauss Law : Electric flux through a Gaussian surface is $1/\epsilon_0$ times the net charge enclosed with in it.

$$\oint \vec{E} \cdot \vec{dS} = \frac{q}{E_0}$$

(iii) Force on the charge at the centre i.e. Charge = $Q/2 = 0$

$$F_A = \frac{1}{4\pi E_0}\frac{2Q \times (Q + {}^{Q}/_{2})}{x^2} = \frac{1}{4\pi E_0}\frac{3Q^2}{x^2}$$

43. (a) Define torque acting on a dipole of dipole moment p placed in a uniform electric field $\vec{E}$. Express it in the vector form and point out the direction along which it acts.
(b) What happens if the field is non-uniform ?

(c) What would happen if the external field $\vec{E}$ is increasing

(i) parallel to $\vec{p}$ and (ii) anti-parallel to $\vec{p}$ **(2016)**

Ans. (a) $\tau = pE \sin\theta$; θ = angle between dipole moment $(\vec{p})$ and electric field $(\vec{E})$

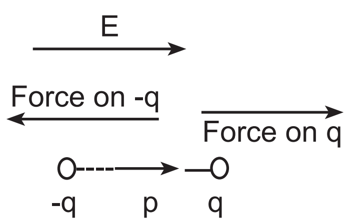
$$\vec{\tau} = \vec{p} \times \vec{E}$$

Direction of torque is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$ given by right hand screw rule.

(b) If the field is non uniform the net force on the dipole will not be zero.

There will be translatory motion of the dipole.

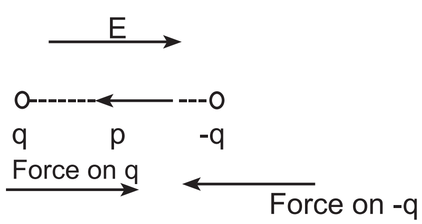
(c) (i) Net force will be in the direction of increasing electric field.



Direction of net force = →

Direction of increasing field = →

(ii) Net force will be in the direction opposite to the increasing field.



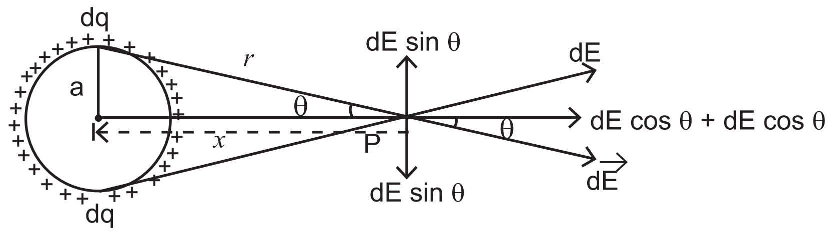
Direction of net force = ←

Direction of increasing field = →

44. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large

distances from the ring, it behaves like a point charge. **(2016)**

Ans.



Net Electric Field at point P = $\int_0^{2\pi a} dE \cos\theta$

dE = Electric Field due to a small element having charge dq

$= \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2}$

Let λ = Linear charge density = $\frac{dq}{dl}$

$\Rightarrow$ $dq = \lambda dl$

Hence E = $\int_0^{2\pi a} \frac{1}{4\pi\varepsilon_0} . \frac{\lambda dl}{r^2}$ x $\frac{x}{r}$, where $\cos\theta = \frac{x}{r}$

$= \frac{\lambda x}{4\pi\varepsilon_0 r^3} (2\pi a)$

$= \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(x^2 + a^2)^{\frac{3}{2}}}$, where total charge $Q = \lambda$ x $2\pi a$

At large distance i.e, x>> a, $E \approx \frac{1}{4\pi\varepsilon_0} . \frac{Q}{x^2}$

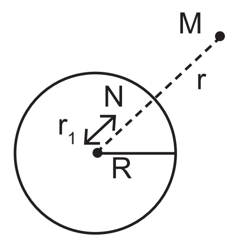
This is the Electric field due to a point charge at distance x.

45. Find the electric field intensity due to a uniformly charged spherical shell at a point (i) outside the shell and (ii) inside the shell. Plot the graph of electric field with distance from the centre of the shell. **(2016)**

Ans. We have by Gauss's law $\oint \vec{E} . \vec{dS} = \frac{Q_{enclosed}}{\varepsilon_0}$

Let Q be the total charge on the shell

(i) For the point M outside the shell, we have



$E. 4\pi r^2 = \frac{Q}{\varepsilon_0}$

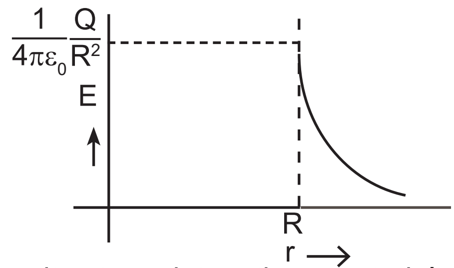
$\therefore E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

(ii) For the point N inside the shell, as charge enclosed inside the shell is zero.

$E. 4\pi r_1^2 = 0$

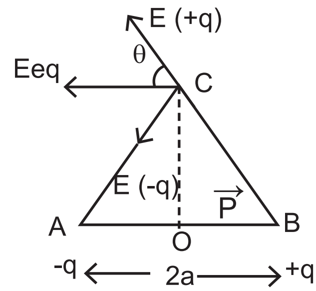
$\therefore E = 0$

The graph is as shown



46. Derive an expression for the electric field intensity at a point on the equatorial line of an electric dipole of dipole moment $\vec{p}$ and length 2a. What is the direction of this field ? **(2016)**

Ans. $E_{+q} = Kq/(r^2 + a^2)$ and $E_{-q} = Kq/(r^2 + a^2)$



The two Electric fields have equal magnitudes and their directions are as shown in diagram

Components along dipole axis get added up while normal components cancel each other.

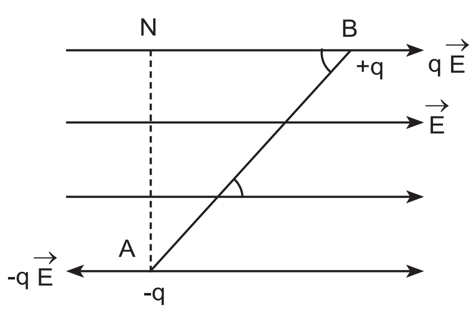
$\therefore E = -[E_{-q} + E_{+q}] \cos\theta\,\hat{r}$

so $E = -\frac{K2qa}{[r^2 + a^2]^{\frac{3}{2}}}\hat{r} = \frac{kp}{[r^2 + a^2]^{\frac{3}{2}}} = \frac{-1}{4\pi\varepsilon_0}\frac{p}{[r^2 + a^2]^{\frac{3}{2}}}$ $\quad (p = 2qa\hat{r})$

$\therefore$ Direction of electric field is opposite to that of dipole moment.

47. An electric dipole of dipole moment $\vec{P}$ is placed in a uniform electric field $\vec{E}$. Obtain the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. **(2015)**

Ans. (i)



The force on charge +q is q $\vec{E}$ and on charge –q is -q $\vec{E}$. These, two parallel forces, acting in the opposite direction, constitute a couple resulting in the torque τ.

Magnitude of torque = $qE \times 2a \sin\theta$

= $PE \sin\theta$ $\quad [\vec{P} = q \times 2\vec{a}]$

= $\vec{P} \times \vec{E}$

(ii) Two pairs of perpendicular vectors:

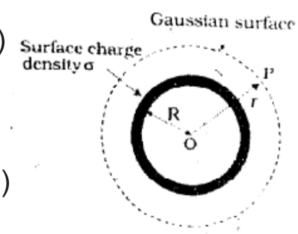
(a) $\vec{\tau}$ is perpendicular to $\vec{P}$ (b) $\vec{\tau}$ is perpendicular to $\vec{E}$

5 MARKS

48. (a) Use Gauss's law to show that due to a uniformly charged spherical shell of radius R, the electric field at any point situated outside the shell at a distance r from its centre is equal to the electric field at the same point, when the entire charge on the shell were concentrated at its centre. Also plot the graph showing the variation of electric field with r, for r ≤ R and r ≥ R. **(2020, 2011)**

(b) Two point charges +1 μC and +4 μC are kept 30 cm apart. How far from the +1 μC charge on the line joining the two charges, will the net electric field be zero ?

Ans. From Gauss's theorem, $\phi = \int \vec{B} \cdot d\vec{S} = \frac{q_m}{\varepsilon_0}$ -----(1)

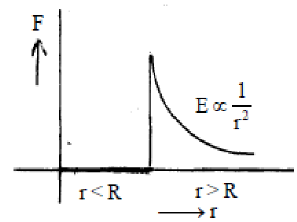


Flux ϕ through $S^{|}$. $\phi = \int \vec{B} \cdot d\vec{S}$

$= \int EdS = E \cdot 4\pi r^2$ -----(2)

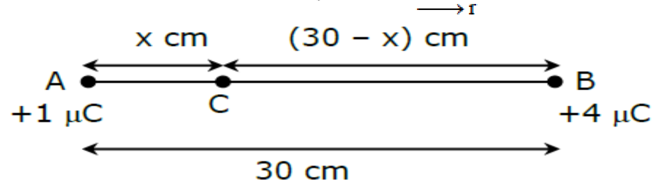
From (1) and (2), $E \cdot 4\pi r^2 = \frac{q_m}{\varepsilon_0}$

$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q_m}{r^2}$



(b) Let at point C, the net electric field is zero.

Electric field at point C,



$E_c = \frac{kq_1}{r_1} - \frac{kq_2}{r_2}$

$E_c = \frac{k\,(1\,\mu C)}{x} - \frac{k\,(4\,\mu C)}{30 - x}$

For $E_c = 0$

$\frac{k\,(1\,\mu C)}{x} = \frac{k\,(4\,\mu C)}{30 - x}$

$\Rightarrow 30 - x = 4x \quad \Rightarrow 5x = 30$

$\therefore x = 6$ cm

At a distance of 6 cm from 1 μC towards 4 μC change, the electric field is zero.

49. (a) Two point charges q_1 and q_2 are kept r distance apart in a uniform external electric field $\vec{E}$. Find the amount of work done in assembling this system of charges.

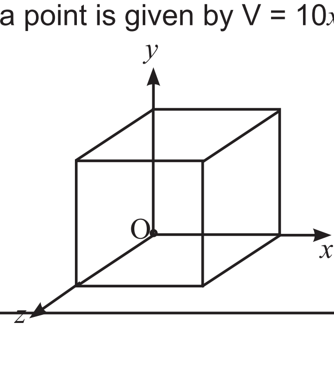
(b) A cube of side 20 cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by $V = 10x + 5$, where V is in volt and x is in m.

Find the

(i) Electric field $\vec{E}$, and

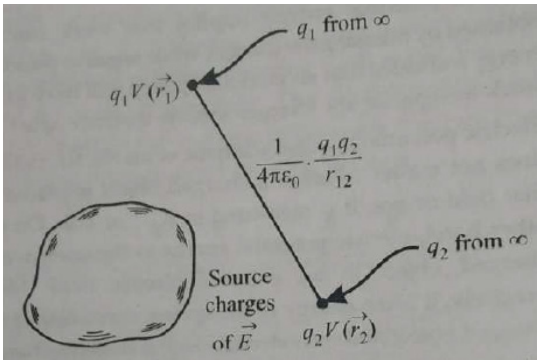
(ii) Total electric flux through the cube.

(2020)



Ans. (a) $V(\vec{r_1})$ and $V(\vec{r_2})$ be the electric potential of the field E at the points having position vectors $\vec{r_1}$ and $\vec{r_2}$ as shown in fig.

Work done in bringing q_1 from ∞ to $\vec{r_1}$ against the external field = $q_1V(\vec{r_1})$



Work done in bringing q_1 from ∞ to $\vec{r_2}$ against the external field = $q_2V(\vec{r_2})$

Work done on q_2 against the force exerted by $q_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_{12}}$

Where r_{12} is the distance between q_1 and q_2.

Total potential energy of the system = The work done in assembling the two charges

Or $\quad U = q_1V(\vec{r_1}) + q_2V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_{12}}.$

(b) (i) Given,

Side of, cube, a = 20 cm

Electric potential, $V = 10x + 5$

Electric field, $E = -\frac{dV}{dx} = -\frac{d(10x+5)}{dx} = -10\ NC^{-1}$

E = -10 NC-1 (along negative X - axis)

(ii) Total electric flux, $\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$

$\phi = \vec{E}\cdot\vec{A_1} + \vec{E}\cdot\vec{A_2} + \vec{E}\cdot\vec{A_3} + \vec{E}\cdot\vec{A_4} + \vec{E}\cdot\vec{A_5} + \vec{E}\cdot\vec{A_6}$

$\phi = -10\ NC^{-1}$ (20 cm x 20 cm) cos 0°

$+ (10\ NC^{-1})$ (20 cm x 20 cm)cos 90°

$+ (-10\ NC^{-1})$ (20 cm x 20 cm) cos 90°

$+ (-10\ NC^{-1})$ (20 cm x 20 cm) cos 90°

$+ (-10\ NC^{-1})$ (20 cm x 20 cm) cos 90°

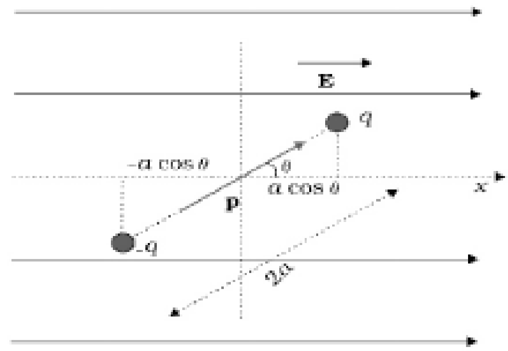
$+ (-10\ NC^{-1})$ (20 cm x 20 cm) cos180°

$\phi = (-10\ NC^{-1})(400\ cm^2)(1) + 0 + 0 + 0 + 0 + (-10\ NC^{-1})(400\ cm^2)(-1) = 0$

50. (a) Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium. **(2019)**

(b) Is the electrostatic potential necessarily zero at a point where the electric field is zero ? Give an example to support your answer.

Ans. (a) Since torque acting on dipole



$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$= pE \sin\theta.\hat{n}$$

Work done $d\omega = \tau.d\theta = pE\sin\theta d\theta$

$$w = \int_{\theta_1}^{\theta_2} dw = pE\int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

$$w = pE[-\cos\theta]_{\theta_1}^{\theta_2} = pE[\cos\theta_1 - \cos\theta_2]$$

if $\theta_1 = 0,\ \theta_2 = \theta$

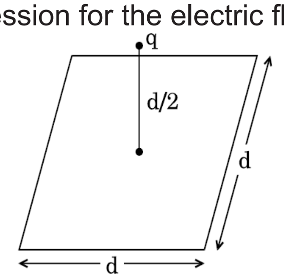
Conditions $w = pE(1 - \cos\theta)$

For stable equilibrium - When electric dipole is parallel to electric field.

For unstable equilibrium -Anti Parallel to electric field.

(b) No. Inside equipotential surface

51. Define electric flux. Is it a scalar or a vector quantity? A point charge q is at a distance of $\frac{d}{2}$ directly above the centre of a square of side d, as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square. **(2018)**

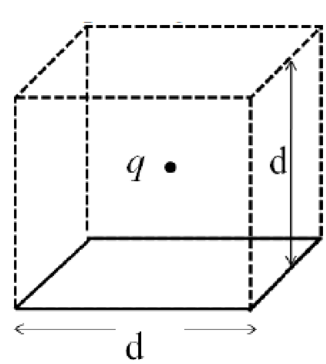


(b) If the point charge is now moved to a distance 'd' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.

Ans. (a) Electric flux through a given surface is defined as the dot product of electric field and area vector over that surface.

$$\phi = \int_s \vec{E} \cdot \vec{ds}$$

It is a scalar quantity



Constructing a cube of side 'd' so that charge 'q' gets placed within of this cube (Gaussian surface)

According to Gauss's law the Electric flux,

$$\varnothing = \frac{\text{Charge enclosed}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

This is the total flux through all the six faces of the cube.

Hence electric flux through the square = $\frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$

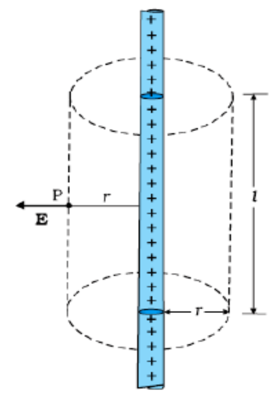
(b) If the charge is moved to a distance d and the side of the square is doubled the cube will be constructed to have a side 2d but the total charge enclosed in it will remain the same. Hence the total flux through the cube and therefore the flux through the square will remain the same as before.

52. (a) Use Gauss' law to derive the expression for the electric field ($\vec{E}$) due to a straight uniformly charged infinite line of charge density λ C/m. **(2018)**

(b) Draw a graph to show the variation of E with perpendicular distance r from the line of charge.

(c) Find the work done in bringing a charge q from perpendicular distance r_1 to r_2 ($r_2 > r_1$).

Ans. (a) To calculate the electric field, imagine a cylindrical Gaussian surface, since the field is everywhere radial, flux through two ends of the cylindrical Gaussian surface is zero.



At cylindrical part of the surface electric field $\vec{E}$ is normal to the surface at every point and its magnitude is constant. Therefore flux through the Gaussian surface

= Flux through the curved cylindrical part of the surface.

= $E \times 2\pi rl$ -------(i)

Applying Gauss's Law Flux, $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

Total charge enclosed = Linear charge density x l = λl

$\therefore \phi = \frac{\lambda l}{\varepsilon_0}$ ------------- (ii)

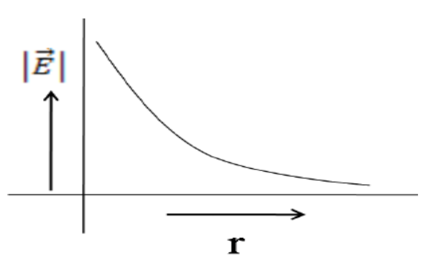
Using Equations (i) & (ii)

$$E \times 2\pi rl = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

In vector notation, $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$ where $\hat{n}$ is a unit vector normal to the line charge

(b) The required graph is as shown:



(a) Work done in moving the charge 'q', through a small displacement 'dr',

$$dW = \vec{F} \cdot \vec{dr}$$

$$dW = q\vec{E} \cdot \vec{dr} = qEdr \cos 0$$

$$dW = q \times \frac{\lambda}{2\pi\varepsilon_0 r} dr$$

Work done in moving the given charge from r_1 to r_2 ($r_2 > r_1$)

$$W = \int_{r_1}^{r_2} dW = \int_{r_1}^{r_2} \frac{\lambda q dr}{2\pi\varepsilon_0 r}$$

$$W = \frac{\lambda q}{2\pi\varepsilon_0}\left[\log_e r_2 - \log_e r_1\right]$$

$$W = \frac{\lambda q}{2\pi\varepsilon_0}\left[\log_e \frac{r_2}{r_1}\right]$$

53. (a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ.

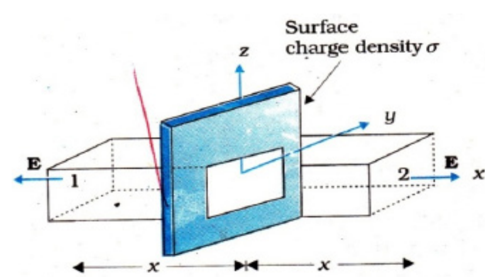
(b) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet. **(2017)**

Ans. (a) $\oint E.\, ds = \frac{q}{\varepsilon_0}$

The electric field E points outwards normal to the sheet. The field lines are parallel to the Gaussian surface except for surfaces 1 and 2.

Hence the net flux = $\oint E.\, ds = EA + EA$ where A is the area of each of the surface 1 and 2.

$$\therefore \quad \oint E.\, ds = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} = 2EA\,;$$



(b) $$W = q\int_{\infty}^{T} \vec{E} \cdot d\vec{r}$$

$$= q\int_{\infty}^{T} (-E\, dr)$$

$$= -q\int_{\infty}^{T} \left(\frac{\sigma}{2\epsilon_0}\right) dr = \frac{q\,\sigma}{2\epsilon} \,|\, \infty - r \,|$$

54. (a) Derive an expression for the electric field E due to a dipole of length '2a' at a point distant r from the centre of the dipole on the axial line.

(b) Draw a graph of E versus r for r >> a.

(c) If this dipole were kept in a uniform external electric field E_0, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases. **(2017)**

Ans. (a)

Electric field at P due to charge (+q) = $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$

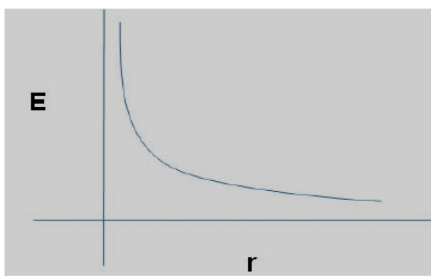
Electric field at P due to charge (-q) = $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$

Net electric Field at P = $E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$

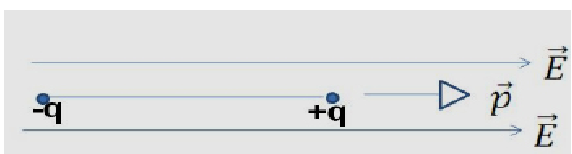
$= \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}$ $\qquad (p = q.2a)$

Its direction is parallel to $\vec{p}$.

(b)



(c)



Stable equilibrium

Unstable equilibrium

(i) For stable Equilibrium: $\vec{p}$ is parallel to $\vec{E}$.

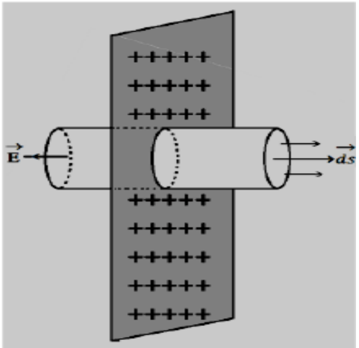
(ii) For unstable equilibrium: $\vec{p}$ is antiparallel to $\vec{E}$

Torque = 0 for (i) as well as case (ii).

55. (i) Use Gauss's law to find the electric field due to a uniformly charged infinite plane sheet. What is the direction of field for positive and negative charge densities ?

(ii) Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same. **(2016)**

Ans. Symmetry of situation suggests that $\vec{E}$ is perpendicular to the plane. Gaussian surface through P like a cylinder of flat caps parallel to the plane and one cap passing through P, the plane being the plane of symmetry for the Gaussian surface.



$\oint \vec{E} \cdot \vec{ds} = \int_{\text{through caps}} \vec{E} \cdot \vec{ds}$

$\vec{E} \perp \vec{ds}$ for all over curved surface and hence $\vec{E}\,\vec{ds} = 0$

$\int_{caps} E\,ds = 2E\Delta s$ where Δs = area of each cap

by Gauss' law $\oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0} = \frac{\sigma \Delta s}{\varepsilon_0}$

$\therefore 2E\Delta s = \frac{\sigma \Delta s}{\varepsilon_0} \quad \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

If σ is positive $\vec{E}$ points normally outwards/away from the sheet

If σ is (-)ve $\vec{E}$ points normally inwards/towards the sheet

$U_s = \frac{1}{2} c_s V_s^2,\ U_p = \frac{1}{2} c_p V_p^2 U$

$\Rightarrow \frac{V_{series}}{V_{parallel}} = \sqrt{\frac{C_{\text{equivalent parallel}}}{C_{\text{equivalent series}}}}$

$= \sqrt{\frac{C_1 + C_2}{\frac{C_1 C_2}{C_1 + C_2}}} = \frac{C_1 + C_2}{\sqrt{C_1 C_2}} = \frac{3}{\sqrt{2}}$

56. (a) Define electric flux. Write its S.I. unit.

"Gauss's law in electrostatics is true for any closed surface, no matter what its shape or size is." Justify this statement with the help of a suitable example.

(b) Use Gauss's law to prove that the electric field inside a uniformly charged spherical shell is zero. **(2015)**

Ans. (a) Total number of electric lines of force passing perpendicular through a given surface.

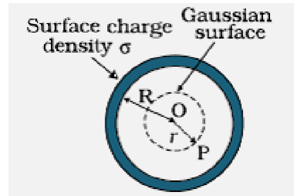
Unit – newton m^2

According to Gauss theorem, the electric flux through a closed surface depends only on the net charge enclosed by the surface and not upon the shape or size of the surface.

For any closed arbitrary slope of the surface enclosing a charge the outward flux is the same as that due to a spherical Gaussian surface enclosing the same charge.

Justification: This is due to the fact

(i) electric field is radial and (ii) the electric field $E \propto \frac{1}{R^2}$



(b) According to Gauss theorem,

$\int \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \cdot q = 0$

(∵ charge inside the shell is zero.)

$\therefore E.dS = 0$, But $dS \neq 0$

$\therefore E = 0$

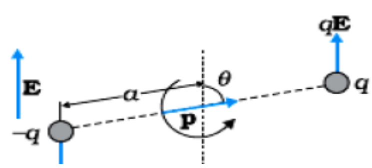
57. (a) Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ placed in a uniform electricfield $\vec{E}$. Depict the direction of the torque. Express it in the vector form.

(b) Show that the potential energy of a dipole making angle θ with the direction of the field is given by $u(\theta) = -\vec{p} \cdot \vec{E}$. Hence find out the amount of work done in rotating it from the position of unstable equilibrium to the stable equilibrium. **(2016)**

Ans. Magnitude of torque = magnitude of either force multiplied by the arm of the couple.

$= qE \times 2a \sin\theta$

$= pE \sin\theta$



Direction of torque is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$.

Vector form $\vec{\tau} = \vec{p} \times \vec{E}$

(b) Work done by external torque in rotating a dipole in uniform electric field is stored as the Potential energy of the system.

$U(\theta_0 \to \theta) = W(\theta_0 \to \theta) = pE(\cos\theta_0 - \cos\theta_1)$

For $\theta_0 = \frac{\pi}{2}$ and $\theta_1 = \theta$

$U(\theta) = pE(\cos\frac{\pi}{2} - \cos\theta) = -pE\cos\theta = -\vec{p} \cdot \vec{E}$

For rotating dipole from position of unstable equilibrium (θ_0 = 180°)to the stable equilibrium (θ = 0°)

$W_{req} = pE(\cos 180° - \cos 0°) = pE(-1 -1) = -2pE$

✧✧✧✧✧