TEACHERS FORUM ®

QUESTION BANK

(solved)

Class XI

MATHEMATICS

SUBJECT EXPERTS

CONTENTS

1 SETS

NCERT SOLUTIONS

EXERCISE 1.1

1. Which of the following are sets ? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world.

Ans. (i) The collection of all months of a year beginning with the letter J is a well-defined collection. So, this collection is a set.

(ii) The collection of ten most talented writers of India is not a well-defined collection because the criteria for determining a writer's talent vary from person to person. So, this collection is not a set.

(iii) A team of eleven best cricket batsmen of the world is not a well-defined collection because the criteria for determining a batsman's talent vary from person to person. So, this collection is not a set.

(iv) The collection of all boys in your class is a well-defined collection. So, this collection is a set.

(v) The collection of all natural numbers less than 100 is a well-defined collection because one can definitely identify a number that belongs to this collection. So, this collection is a set.

(vi) A collection of novels written by the writer Munshi Prem Chand is a well-defined collection. So, this collection is a set.

(vii) The collection of all even integers is a well-defined collection. So, this collection is a set.

(viii) The collection of questions in this chapter is a well-defined collection. So, this collection is a set.

(ix) The collection of most dangerous animals of the world is not a well- defined collection because the criteria for determining the dangerousness of an animal vary from person to person. So, this collection is not a set.

2. Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

(i) 5. . .A (ii) 8 . . . A (iii) 0. . .A (iv) 4. . . A (v) 2. . .A (vi) 10. . .A

Ans. (i) $5 \in A$ (ii) $8 \notin A$ (iii) $0 \notin A$

(iv) $4 \in A$ (v) $2 \in A$ (vi) $10 \notin A$

3. Write the following sets in roster form:

(i) A = {x : x is an integer and $-3 < x < 7$}

(ii) B = {x : x is a natural number less than 6}

(iii) C = {x : x is a two-digit natural number such that the sum of its digits is 8}

(iv) D = {x : x is a prime number which is divisor of 60}

(v) E = The set of all letters in the word TRIGONOMETRY

(vi) F = The set of all letters in the word BETTER

Ans. (i) The elements of this set are –2, –1, 0, 1, 2, 3, 4, 5, and 6 only.

So A = {–2, –1, 0, 1, 2, 3, 4, 5, 6}

(ii) The elements of this set are 1, 2, 3, 4, and 5 only.

So B = {1, 2, 3, 4, 5}

(iii) The elements of this set are 17, 26, 35, 44, 53, 62, 71, and 80 only.

So C = {17, 26, 35, 44, 53, 62, 71, 80}

(iv) $60 = 2 \times 2 \times 3 \times 5$

$\Rightarrow$ The elements of this set are 2, 3, and 5 only.

So, D = {2, 3, 5}.

(v) E = {T, R, I, G, O, N, M, E, Y}

(vi) F = {B, E, T, R}

4. Write the following sets in the set-builder form :

(i) (3, 6, 9, 12} (ii) {2,4,8,16,32} (iii) {5, 25, 125, 625}

(iv) {2, 4, 6, . . .} (v) {1,4,9, . . .,100}

Ans. (i) {3, 6, 9, 12} = {x : x = 3n, n $\in$ N and $1 \le n \le 4$}

(ii) $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$, and $32 = 2^5$.

$\therefore$ {2, 4, 8, 16, 32} = {x : x = 2n, n $\in$ N and 1 ≤ n ≤ 5}

(iii) $5 = 5^1$, $25 = 5^2$, $125 = 5^3$, and $625 = 5^4$.

$\therefore$ {5, 25, 125, 625} = {x : x = 5n, n $\in$ N and 1 ≤ n ≤ 4}

(iv) It is a set of all even natural numbers.

$\therefore$ {2, 4, 6 ...} = {x : x is an even natural number}

(v) It can be seen that $1 = 1^2$, $4 = 2^2$, $9 = 3^2$ ...$100 = 10^2$.

$\therefore$ {1, 4, 9... 100} = {x : $x = n^2$, n $\in$ N and 1 ≤ n ≤ 10}

5. List all the elements of the following sets :

(i) A = {x : x is an odd natural number}

(ii) B = {x : x is an integer, $-\frac{1}{2} < x < \frac{9}{2}$}

(iii) C = {x : x is an integer, $x^2 \le 4$}

(iv) D = {x : x is a letter in the word "LOYAL"}

(v) E = {x : x is a month of a year not having 31 days}

(vi) F = {x : x is a consonant in the English alphabet which precedes k }.

Ans. (i) A = {1, 3, 5, 7, 9 ...}

(ii) $-\frac{1}{2} = -0.5$ and $\frac{9}{2} = 4.5$

$\therefore$ B = {0, 1, 2, 3, 4}

(iii) $(-1)^2 = 1 \le 4$; $(-2)^2 = 4 \le 4$; $(-3)^2 = 9 > 4$

$0^2 = 0 \le 4$; $1^2 = 1 \le 4$; $2^2 = 4 \le 4$; $3^2 = 9 > 4$

$\therefore$ C = {–2, –1, 0, 1, 2}

(iv) D = {L, O, Y, A}

(v) E = {February, April, June, September, November}

(vi) F = {b, c, d, f, g, h, j}

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) {1, 2, 3, 6}	(a) {x : x is a prime number and a divisor of 6}
(ii) {2, 3}	(b) {x : x is an odd natural number less than 10}
(iii) {M,A,T,H,E,I,C,S}	(c) {x : x is natural number and divisor of 6}
(iv) {1, 3, 5, 7, 9}	(d) {x : x is a letter of the word MATHEMATICS}.

Ans. (i) (i) → (c). (ii) (ii) → (a).

(iii) (iii) → (d). (iv) (iv) → (b).

EXERCISE 1.2

1. Which of the following are examples of the null set

(i) Set of odd natural numbers divisible by 2

(ii) Set of even prime numbers

(iii) {x : x is a natural numbers, x < 5 and x > 7 }

(iv) {y : y is a point common to any two parallel lines}

Ans. (i) A set of odd natural numbers divisible by 2 is a null set because no odd number is divisible by 2.

(ii) A set of even prime numbers is not a null set because 2 is an even prime number.

(iii) {x : x is a natural number, x < 5 and x > 7} is a null set because a number cannot be simultaneously less than 5 and greater than 7.

(iv) {y : y is a point common to any two parallel lines} is a null set because parallel lines do not intersect.

2. Which of the following sets are finite or infinite

(i) The set of months of a year

(ii) {1, 2, 3 ...}

(iii) {1, 2, 3 ... 99, 100}

(iv) The set of positive integers greater than 100

(v) The set of prime numbers less than 99

Ans. (i) The set is a finite set because it has 12 elements.

(ii) {1, 2, 3 ...} is an infinite set as it has infinite number of natural numbers.

(iii) {1, 2, 3 ...99, 100} is a finite set.

(iv) The set of positive integers greater than 100 is an infinite set.

(v) The set of prime numbers less than 99 is a finite set.

3. State whether each of the following set is finite or infinite:

(i) The set of lines which are parallel to the x-axis

(ii) The set of letters in the English alphabet

(iii) The set of numbers which are multiple of 5

(iv) The set of animals living on the earth

(v) The set of circles passing through the origin (0, 0)

Ans. (i) The set of lines which are parallel to the x-axis is an infinite set.

(ii) The set of letters in the English alphabet is a finite set because it has 26 elements.

(iii) The set of numbers which are multiple of 5 is an infinite set.

(iv) The set of animals living on the earth is a finite set.

(v) The set of circles passing through the origin (0, 0) is an infinite set.

4. In the following, state whether A = B or not:

(i) A = {a, b, c, d}; B = {d, c, b, a}

(ii) A = {4, 8, 12, 16}; B = {8, 4, 16, 18}

(iii) A = {2, 4, 6, 8, 10}; B = {x : x is positive even integer and $x \leq 10$}

(iv) A = {x : x is a multiple of 10}; B = {10, 15, 20, 25, 30 ...}

Ans. (i) The order in which the elements of a set are listed is not significant.

$\therefore$ A = B

(ii) Here $12 \in A$ but $12 \notin B$.

$\therefore A \neq B$

(iii) A = {2, 4, 6, 8, 10}

B = {2, 4, 6, 8, 10}

$\therefore$ A = B

(iv) A = {x : x is a multiple of 10}

B = {10, 15, 20, 25, 30 ...}

Here $15 \in B$ but $15 \notin A$.

$\therefore A \neq B$

5. Are the following pair of sets equal? Give reasons.

(i) A = {2, 3}; B = {x : x is solution of $x^2 + 5x + 6 = 0$}

(ii) A = {x : x is a letter in the word FOLLOW}; B = {y : y is a letter in the word WOLF}

Ans. (i) $x^2 + 5x + 6 = 0$

` $x(x + 3) + 2(x + 3) = 0$

$(x + 2)(x + 3) = 0$

$\Rightarrow x = -2$ or $x = -3$

$\Rightarrow$ B = {–2, –3}

$\therefore A \neq B$

(ii) A = {F, O, L, W}

B = {W, O, L, F}

$\therefore$ A = B

6. From the sets given below, select equal sets:

A = {2, 4, 8, 12}, B = {1, 2, 3, 4}, C = {4, 8, 12, 14}, D = {3, 1, 4, 2}

E = {–1, 1}, F = {0, a}, G = {1, –1}, H = {0, 1}

Ans. The order in which the elements of a set are listed is not significant.

So, B = D and E = G.

EXERCISE 1.3

1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:

(i) {2, 3, 4} ... {1, 2, 3, 4, 5}

(ii) {a, b, c} ... {b, c, d}

(iii) {x : x is a student of Class XI of your school} ... {x : x student of your school}

(iv) {x : x is a circle in the plane} ... {x : x is a circle in the same plane with radius 1 unit}

(v) {x : x is a triangle in a plane}...{x : x is a rectangle in the plane}

(vi) {x : x is an equilateral triangle in a plane}... {x : x is a triangle in the same plane}

(vii) {x : x is an even natural number} ... {x : x is an integer}

Ans. (i) {2, 3, 4} $\subset$ {1, 2, 3, 4, 5}

(ii) {a, b, c} $\not\subset$ {b, c, d}

(iii) {x : x is a student of class XI of your school} $\subset$ {x : x is student of your school}

(iv) {x : x is a circle in the plane} $\not\subset$ {x : x is a circle in the same plane with radius 1 unit}

(v) {x : x is a triangle in a plane} $\not\subset$ {x : x is a rectangle in the plane}

(vi) {x : x is an equilateral triangle in a plane} $\subset$ {x : x in a triangle in the same plane}

(vii) {x : x is an even natural number} $\subset$ {x : x is an integer}

2. Examine whether the following statements are true or false:

(i) {a, b} $\not\subset$ {b, c, a}

(ii) {a, e} $\subset$ {x : x is a vowel in the English alphabet}

(iii) {1, 2, 3} $\subset$ {1, 3, 5}

(iv) {a} $\subset$ {a. b, c}

(v) {a} $\in$ (a, b, c)

(vi) {x : x is an even natural number less than 6} $\subset$ {x: x is a natural number which divides 36}

Ans. (i) False. Each element of {a, b} is also an element of {b, c, a}.

(ii) True. a, e are two vowels of the English alphabet.

(iii) False. 2 $\in$ {1, 2, 3}; however, 2 $\notin$ {1, 3, 5}

(iv) True. Each element of {a} is also an element of {a, b, c}.

(v) False. The elements of {a, b, c} are a, b, c. So, {a} $\subset$ {a, b, c}

(vi) True. {x : x is an even natural number less than 6} = {2, 4}

{x : x is a natural number which divides 36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}

3. Let A = {1, 2, {3, 4,}, 5}. Which of the following statements are incorrect and why?

(i) {3, 4} $\subset$ A (ii) {3, 4}} $\in$ A (iii) {{3, 4}} $\subset$ A

(iv) 1 $\in$ A (v) 1 $\subset$ A (vi) {1, 2, 5} $\subset$ A

(vii) {1, 2, 5} $\in$ A (viii) {1, 2, 3} $\subset$ A (ix) Φ $\in$ A

(x) Φ $\subset$ A (xi) {Φ} $\subset$ A

Ans. A = {1, 2, {3, 4}, 5}

(i) The statement {3, 4} $\subset$ A is incorrect because 3 $\in$ {3, 4}; however, 3 $\notin$ A.

(v) The statement 1 $\subset$ A is incorrect because an element of a set can never be a subset of itself.

(vii) The statement {1, 2, 5} $\in$ A is incorrect because {1, 2, 5} is not an element of A.

(viii) The statement {1, 2, 3} $\subset$ A is incorrect because 3 $\in$ {1, 2, 3}; however,3 $\notin$ A.

(ix) The statement Φ $\in$ A is incorrect because Φ is not an element of A.

(xi) The statement {Φ} $\subset$ A is incorrect because Φ $\in$ {Φ}; however, Φ $\in$ A.

4. Write down all the subsets of the following sets:

(i) {a} (ii) {a, b} (iii) {1, 2, 3} (iv) Φ

Ans. (i) Φ and {a}. (ii) Φ, {a}, {b}, and {a, b}.

(iii) Φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3} and {1, 2, 3} (iv) Φ.

5. How many elements has P(A), if A = Φ?

Ans. If A = Φ, then n(A) = 0.

$\therefore$ n[P(A)] = 2^0 = 1

So, P(A) has one element.

6. Write the following as intervals:

(i) $\{x : x \in R, -4 < x \leq 6\}$ (ii) $\{x : x \in R, -12 < x < -10\}$

(iii) $\{x : x \in R, 0 \leq x < 7\}$ (iv) $\{x : x \in R, 3 \leq x \leq 4\}$

Ans. (i) $(-4, 6]$ (ii) $(-12, -10)$ (iii) $[0, 7)$ (iv) $[3, 4]$

7. Write the following intervals in set-builder form:

(i) $(-3, 0)$ (ii) $[6, 12]$ (iii) $(6, 12]$ (iv) $[-23, 5)$

Ans. (i) $(-3, 0) = \{x : x \in R, -3 < x < 0\}$ (ii) $[6, 12] = \{x : x \in R, 6 \leq x \leq 12\}$

(iii) $(6, 12] = \{x : x \in R, 6 < x \leq 12\}$ (iv) $[-23, 5) = \{x : x \in R, -23 \leq x < 5\}$

8. What universal set (s) would you propose for each of the following?

(i) The set of right triangles

(ii) The set of isosceles triangles

Ans. (i) the set of triangles or the set of polygons.

(ii) the set of triangles or the set of polygons.

9. Given the sets A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}, which of the following may be considered as universals set (s) for all the three sets A, B and C

(i) {0, 1, 2, 3, 4, 5, 6} (ii) Φ

(iii) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} (iv) {1, 2, 3, 4, 5, 6, 7, 8}

Ans. (i) Here $A \subset \{0, 1, 2, 3, 4, 5, 6\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6\}$

$C \not\subset \{0, 1, 2, 3, 4, 5, 6\}$

So, the set {0, 1, 2, 3, 4, 5, 6} cannot be the universal set for the sets A, B, and C.

(ii) $A \not\subset \Phi$, $B \not\subset \Phi$, $C \not\subset \Phi$

So, Φ cannot be the universal set for the sets A, B, and C.

(iii) $A \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$C \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

So, the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} is the universal set for the sets A, B, and C.

(iv) $A \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

$C \not\subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

So, the set {1, 2, 3, 4, 5, 6, 7, 8} cannot be the universal set for the sets A, B, and C.

EXERCISE 1.4

1. Find the union of each of the following pairs of sets:

(i) X = {1, 3, 5}; Y = {1, 2, 3}

(ii) A = {a, e, i, o, u}; B = {a, b, c}

(iii) A = $\{x : x$ is a natural number and multiple of 3}

B = $\{x : x$ is a natural number less than 6}

(iv) A = $\{x : x$ is a natural number and $1 < x \leq 6\}$

B = $\{x : x$ is a natural number and $6 < x < 10\}$

(v) A = {1, 2, 3}; B = Φ

Ans. (i) $X \cup Y = \{1, 2, 3, 5\}$

(ii) $A \cup B = \{a, b, c, e, i, o, u\}$

(iii) A = {3, 6, 9 ...}

B = {1, 2, 3, 4, 5, 6}

$A \cup B = \{x : x = 1, 2, 4, 5$ or a multiple of 3}

(iv) A = {2, 3, 4, 5, 6}

B = {7, 8, 9}

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore A \cup B = \{x : x \in N \text{ and } 1 < x < 10\}$

(v) $A \cup B = \{1, 2, 3\}$

2. Let A = {a, b}, B = {a, b, c}. Is $A \subset B$? What is $A \cup B$?

Ans. A = {a, b} and B = {a, b, c}

Yes, $A \subset B$.

$A \cup B = \{a, b, c\} = B$

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Ans. $A \subset B$, then $A \cup B = B$.

4. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}; find

(i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$ (iv) $B \cup D$

(v) $A \cup B \cup C$ (vi) $A \cup B \cup D$ (vii) $B \cup C \cup D$

Ans. (i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

5. Find the intersection of each pair of sets:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

(iii) $A = \{x : x$ is a natural number and multiple of 3$\}$

$B = \{x : x$ is a natural number less than 6$\}$

(iv) $A = \{x : x$ is a natural number and $1 < x \leq 6\}$

$B = \{x : x$ is a natural number and $6 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \Phi$

Ans. (i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$

$X \cap Y = \{1, 3\}$

(ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

$A \cap B = \{a\}$

(iii) $A = (3, 6, 9 ...\}$

$B = \{1, 2, 3, 4, 5\}$

$\therefore A \cap B = \{3\}$

(iv) $A = \{2, 3, 4, 5, 6\}$

$B = \{7, 8, 9\}$

$A \cap B = \Phi$

(v) $A = \{1, 2, 3\}$, $B = \Phi$. So, $A \cap B = \Phi$

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$ (ii) $B \cap C$ (iii) $A \cap C \cap D$

(iv) $A \cap C$ (v) $B \cap D$ (vi) $A \cap (B\ C)$

(vii) $A \cap D$ (viii) $A \cap (B\ D)$ (ix) $(A \cap B) \cap (B\ C)$

(x) (A D) ∩ (B C)

Ans. (i) A ∩ B = {7, 9, 11} (ii) B ∩ C = {11, 13}

(iii) A ∩ C ∩ D = { A ∩ C} ∩ D = {11} ∩ {15, 17} = Φ

(iv) A ∩ C = {11} (v) B ∩ D = Φ

(vi) A ∩ (B C) = (A ∩ B) (A ∩ C) = {7, 9, 11} {11} = {7, 9, 11}

(vii) A ∩ D = Φ

(viii) A ∩ (B D) = (A ∩ B) (A ∩ D) = {7, 9, 11} Φ = {7, 9, 11}

(ix) (A ∩ B) ∩ (B C) = {7, 9, 11} ∩ {7, 9, 11, 13, 15} = {7, 9, 11}

(x) (A D) ∩ (B C) = {3, 5, 7, 9, 11, 15, 17) ∩ {7, 9, 11, 13, 15} = {7, 9, 11, 15}

7. If A = {x : x is a natural number}, B ={x : x is an even natural number}

C = {x : x is an odd natural number} and D = {x : x is a prime number}, find

(i) A ∩ B (ii) A ∩ C (iii) A ∩ D

(iv) B ∩ C (v) B ∩ D (vi) C ∩ D

Ans. A = {1, 2, 3, 4, 5 ...} B = {2, 4, 6, 8 ...}

C = {1, 3, 5, 7, 9 ...} D = {2, 3, 5, 7 ...}

(i) A ∩B = {x : x is a even natural number} = B

(ii) A ∩ C = {x : x is an odd natural number} = C

(iii) A ∩ D = {x : x is a prime number} = D

(iv) B ∩ C = Φ

(v) B ∩ D = {2}

(vi) C ∩ D = {x : x is odd prime number}

8. Which of the following pairs of sets are disjoint

(i) {1, 2, 3, 4} and {x : x is a natural number and $4 \leq x \leq 6$}

(ii) {a, e, i, o, u} and {c, d, e, f}

(iii) {x : x is an even integer} and {x : x is an odd integer}

Ans. (i) {1, 2, 3, 4}

{x : x is a natural number and $4 \leq x \leq 6$} = {4, 5, 6}

Now, {1, 2, 3, 4} ∩ {4, 5, 6} = {4}

So, this pair of sets is not disjoint.

(ii) {a, e, i, o, u} ∩ (c, d, e, f} = {e}

So, {a, e, i, o, u} and (c, d, e, f} are not disjoint.

(iii) $\{x : x$ is an even integer$\} \cap \{x : x$ is an odd integer$\} = \Phi$

So, this pair of sets is disjoint.

9. If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20},

C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}; find

(i) A – B (ii) A – C (iii) A – D

(iv) B – A (v) C – A (vi) D – A

(vii) B – C (viii) B – D (ix) C – B

(x) D – B (xi) C – D (xii) D – C

Ans. (i) A – B = {3, 6, 9, 15, 18, 21} (ii) A – C = {3, 9, 15, 18, 21}

(iii) A – D = {3, 6, 9, 12, 18, 21} (iv) B – A = {4, 8, 16, 20}

(v) C – A = {2, 4, 8, 10, 14, 16} (vi) D – A = {5, 10, 20}

(vii) B – C = {20} (viii) B – D = {4, 8, 12, 16}

(ix) C – B = {2, 6, 10, 14} (x) D – B = {5, 10, 15}

(xi) C – D = {2, 4, 6, 8, 12, 14, 16} (xii) D – C = {5, 15, 20}

10. If X = {a, b, c, d} and Y = {f, b, d, g}, find

(i) X – Y (ii) Y – X (iii) X ∩ Y

Ans. (i) X – Y = {a, c} (ii) Y – X = {f, g} (iii) X ∩ Y = {b, d}

11. If R is the set of real numbers and Q is the set of rational numbers, then what is R – Q?

Ans. R: set of real numbers

Q: set of rational numbers

Therefore, R – Q is a set of irrational numbers.

12. State whether each of the following statement is true or false. Justify your

(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.

(ii) {a, e, i, o, u } and {a, b, c, d} are disjoint sets.

(iii) {2, 6, 10, 14} and {3, 7, 11, 15} are disjoint sets.

(iv) {2, 6, 10} and {3, 7, 11} are disjoint sets.

Ans. (i) False

$3 \in \{2, 3, 4, 5\}$, and $3 \in \{3, 6\}$

$\Rightarrow \{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

(ii) False

a $\in$ {a, e, i, o, u}, and a $\in$ {a, b, c, d}

$\Rightarrow$ {a, e, i, o, u } $\cap$ {a, b, c, d} = {a}

(iii) True, {2, 6, 10, 14} $\cap$ {3, 7, 11, 15} = Φ

(iv) True, {2, 6, 10} $\cap$ {3, 7, 11} = Φ

EXERCISE 1.5

1. Let U = {1, 2, 3; 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find

(i) A^{l} (ii) B^{l} (iii) $(A \cup C)^{l}$

(iv) $(A \cup B)^{l}$ (v) $(A^{l})^{l}$ (vi) $(B - C)^{l}$

Ans. (i) A^{l} = {5, 6, 7, 8, 9} (ii)B^{l} = {1, 3, 5, 7, 9}

(iii) $A \cup C$ = {1, 2, 3, 4, 5, 6}

$\therefore (A \cup C)^{l}$ = {7, 8, 9}

(iv) $A \cup B$ = {1, 2, 3, 4, 6, 8}

$(A \cup B)^{l}$ = {5, 7, 9}

(v) $(A^{l})^{l}$ = A = {1, 2, 3, 4}

(vi) B - C = {2, 8}

$\therefore$ $(B - C)^{l}$ = {1, 3, 4, 5, 6, 7, 9}

2. If U = {a, b, c, d, e, f, g, h}, find the complements of the following sets:

(i) A = {a, b, c} (ii) B = {d, e, f, g}

(iii) C = {a, c, e, g} (iv) D = {f, g, h, a}

Ans. (i) A = {a, b, c} $\therefore$ A^{l} = {d, e, f, g, h}

(ii) B = {d, e, f, g} $\therefore B^{l}$ = {a, b, c, h}

(iii) C = {a, c, e, g} $\therefore C^{l}$ = {b, d, f, h}

(iv) D = {f, g, h, a} $\therefore D^{l}$ = {b, c, d, e}

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) {x : x is an even natural number} (ii) {x : x is an odd natural number}

(iii) {x : x is a positive multiple of 3} (iv) {x : x is a prime number}

(v) {x : x is a natural number divisible by 3 and 5} (vi) {x : x is a perfect square}

(vii) {x : x is perfect cube} (viii) {x : x + 5 = 8}

(ix) $\{x : 2x + 5 = 9\}$ (x) $\{x : x \geq 7\}$

(xi) $\{x : x$ N and $2x + 1 > 10\}$

Ans. U = N: Set of natural numbers

(i) $\{x : x$ is an even natural number$\}' = \{x : x$ is an odd natural number$\}$

(ii) {x: x is an odd natural number}$'$ = $\{x : x$ is an even natural number$\}$

(iii) $\{x : x$ is a positive multiple of 3$\}' = \{x : x$ N and x is not a multiple of 3$\}$

(iv) $\{x : x$ is a prime number$\}' = \{x : x$ is a positive composite number and $x = 1\}$

(v) $\{x : x$ is a natural number divisible by 3 and 5$\}' = \{x : x$ is a natural number that is not divisible by 3 or 5$\}$

(vi) $\{x : x$ is a perfect square$\}' = \{x : x$ N and x is not a perfect square$\}$

(vii) $\{x : x$ is a perfect cube$\}' = \{x : x$ N and x is not a perfect cube$\}$

(viii) $\{x : x + 5 = 8\}' = \{x : x$ N and $x \neq 3\}$

(ix) $\{x : 2x + 5 = 9\}' = \{x : x$ N and $x \neq 2\}$

(x) $\{x : x \geq 7\}' = \{x : x$ N and $x < 7\}$

(xi) $\{x : x$ N and $2x + 1 > 10\}' = \{x : x$ N and $x \leq 9/2\}$

4. If U = {1, 2, 3, 4, 5,6,7,8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}.

Verify that : (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Ans. (i) $(A \cup B)' = \{2, 3, 4, 5, 6, 7, 8\}' = \{1, 9\}$

$A' \cap B' = \{1, 3, 5, 7, 9\} \cap (1, 4, 6, 8, 9) = \{1, 9\}$

$\therefore (A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} = \{1, 3, 4, 5, 6, 7, 8, 9\}$

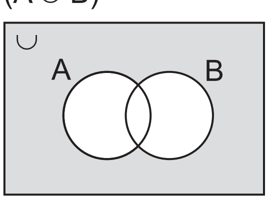
$\therefore (A \cap B)' = A' \cup B'$

5. Draw appropriate Venn diagram for each of the following:

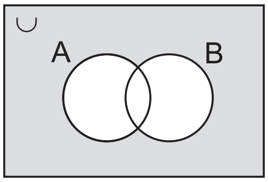
(i) $(A \cup B)'$ (ii) $A' \cap B'$

(iii) $(A \cap B)'$ (iv) $A' \cup B'$

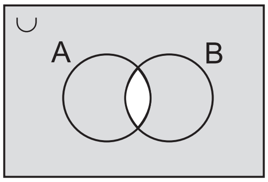
Ans. (i) $(A \cup B)'$



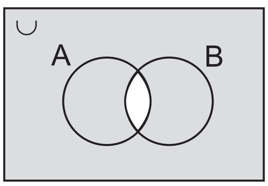
(ii) $A' \cap B'$



(iii) $(A \cap B)'$



(iv) $A' \cup B'$



6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A'?

Ans. A' is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = ...$ (ii) $\Phi' \cap A = ...$ (iii) $A \cap A' = ...$ (iv) $U' \cup A = ...$

Ans. (i) $A \cup A' = U$

(ii) $\Phi' \cap A = U \cap A = A$

$\therefore \Phi' \cap A = A$

(iii) $A \cap A' = \Phi$

(iv) $U' \cup A = \Phi \cap A$

$\therefore U' \cup A = \Phi$

EXERCISE 1.6

1. If X and Y are two sets such that n(X) = 17, n(Y) = 23 and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Ans. Given, $n(X) = 17, n(Y) = 23, n(X \cup Y) = 38$

$n(X \cap Y) = ?$

We know that: $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\therefore 38 = 17 + 23 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 40 - 38 = 2$

$\therefore n(X \cap Y) = 2$

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?

Ans. Given, $n(X \cup Y) = 18, n(X) = 8, n(Y) = 15, n(X \cap Y) = ?$

We know that, $n(X \cup Y) = n(X) = n(Y) - n(X \cap Y)$

$\therefore 18 = 18 + 15 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 23 - 18 = 5$

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Ans. Given, $n(H \cup E) = 400, n(H) = 250, n(E) = 200\ n(H \cap E) = ?$

We know that, $n(H \cup E) = n(H) + n(E) – n(H \cap E)$

$\therefore 400 = 250 + 200 – n(H \cap E)$

$\Rightarrow 400 = 450 – n(H \cap E)$

$\Rightarrow n(H \cap E) = 450 – 400 = 50$

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

Ans. Given, $n(S) = 21, n(T) = 32, n(S \cap T) = 11$

We know that; $n(S \cup T) = n(S) + n(T) – n(S \cap T)$

$\therefore n(S \cup T) = 21 + 32 – 11 = 42$

5. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Ans. Given, $n(X) = 40, n(X \cup Y) = 60, n(X \cap Y) = 10$

We know that, $n(X \cup Y) = n(X) + n(Y) – n(X \cap Y)$

$\therefore 60 = 40 + n(Y) – 10$

$\Rightarrow n(Y) = 60 – (40 – 10) = 30$

6. In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

Ans. Given, $n(C \cup T) = 70, n(C) = 37, n(T) = 52$

We know that, $n(C \cup T) = n(C) + n(T) – n(C \cap T)$

$\therefore 70 = 37 + 52 – n(C \cap T)$

$\Rightarrow 70 = 89 – n(C \cap T)$

$\Rightarrow n(C \cap T) = 89 - 70 = 19$

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Ans. Given, $n(C \cup T) = 65, n(C) = 40, n(C \cap T) = 10$

We know that: $n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore 65 = 40 + n(T) - 10$

$\Rightarrow 65 = 30 + n(T)$

$\Rightarrow n(T) = 65 - 30 = 35$

Therefore, 35 people like tennis.

Now, $(T - C) \cup (T \cap C) = T$

Also, $(T - C) \cap (T \cap C) = \Phi$

$\therefore n(T) = n(T - C) + n(T \cap C)$

$\Rightarrow 35 = n(T - C) + 10$

$\Rightarrow n(T - C) = 35 - 10 = 25$

Thus, 25 people like only tennis.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Ans. Given, $n(F) = 50, n(S) = 20, n(S \cap F) = 10$

We know that: $n(S \cup F) = n(S) + n(F) - n(S \cap F)$

$= 20 + 50 - 10 = 60$

So, 60 people in the committee speak at least one of the two languages.

MISCELLANEOUS QUESTIONS

1. Decide, among the following sets, which sets are subsets of one and another:

$A = \{x : x \in R$ and x satisfy $x^2 - 8x + 12 = 0\}$, $B = \{2, 4, 6\}$, $C = \{2, 4, 6, 8...\}$, $D = \{6\}$.

Ans. $x^2 - 8x + 12 = 0$

$\Rightarrow x = 2, 6$

$\therefore A = \{2, 6\}$

$B = \{2, 4, 6\}$, $C = \{2, 4, 6, 8 ...\}$, $D = \{6\}$

$\therefore D \subset A \subset B \subset C$

Hence, $A \subset B$, $A \subset C$, $B \subset C$, $D \subset A$, $D \subset B$, $D \subset C$

2. In each of the following, determine whether the statement is true or false. If it is true,

prove it. If it is false, give an example.

(i) If $x \in$ A and A $\in$ B, then $x \in$ B (ii) If A $\subset$ B and B $\in$ C, then A $\in$ C

(iii) If A $\subset$ B and B $\in$ C, then A $\in$ C (iv) If A $\not\subset$ B and B $\not\subset$ C, then A $\not\subset$ C

(v) If $x \in$ A and A $\not\subset$ B, then $x \in$ B (vi) If A $\subset$ B and $x \notin$ B, then $x \notin$ A

Ans. (i) False (ii) False (iii) True

(iv) False (v) False (vi) True

3. Let A, B and C be the sets such that A $\cup$ B = A $\cup$ C and A $\cap$ B = A $\cap$ C. show that B = C.

Ans. Let, A, B and C be the sets such that A $\cup$ B = A $\cup$ C and A $\cap$ B = A $\cap$ C.

To show: B = C

Let $x \in$ B

$\Rightarrow x \in$ A $\cup$ B [B $\subset$ A $\cup$ B]

$\Rightarrow x \in$ A or $x \in$ B [A $\cup$ B = A $\cup$ C]

Case I $x \in$ A

Also, $x \in$ B

$\therefore x \in$ A $\cap$ B

$\Rightarrow x \in$ A $\cap$ C [$\because$ A $\cap$ B = A $\cap$ C]

$\therefore x \in$ A and $x \in$ C

$\therefore x \in$ C

$\therefore$ B $\subset$ C

Similarly, we can show that C $\subset$ B.

$\therefore$ B = C

4. Show that the following four conditions are equivalent:

(i) A $\subset$ B (ii) A – B = Φ (iii) A $\cup$ B = B (iv) A $\cap$ B = A

Ans. First, we have to show that (i) $\Leftrightarrow$ (ii).

Let A $\subset$ B

To show: A – B ≠ Φ

If possible, suppose A – B ≠ Φ

This means that there exists $x \in$ A, $x \neq$ B, which is not possible as A $\subset$ B.

$\therefore$ A – B = Φ

$\therefore$ A $\subset$ B $\Rightarrow$ A – B = Φ

$$\text{Let } A - B = \Phi$$

To show: $A \subset B$

Let $x \in A$

Clearly, $x \in B$ because if $x \notin B$, then $A - B \neq \Phi$

$$\therefore A - B = \Phi \Rightarrow A \subset B$$

$\Rightarrow$ (i) $\Leftrightarrow$ (ii)

Let $A \subset B$

To show: $A \cup B = B$

Clearly, $B \subset A \cup B$

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

Case I: $x \in A$

$\Rightarrow x \in B$ $[\because A \subset B]$

$\therefore A \cup B \subset B$

Case II: $x \in B$

Then, $A \cup B = B$

Conversely, let $A \cup B = B$

Let $x \in A$

$\Rightarrow x \in A \cup B$ $[\because A \subset A \cup B]$

$\Rightarrow x \in B$ $[\because A \subset B = B]$

$\therefore A \subset B$

Hence, (i) $\Leftrightarrow$ (iii)

Now, we have to show that (i) $\Leftrightarrow$ (iv).

Let $A \subset B$

Clearly $A \cap B \subset A$

Let $x \in A$

We have to show that $x \in A \cap B$

As $A \subset B$, x B

$\therefore x \in A \cap B$

$\therefore A \subset A \cap B$

Hence, $A = A \cap B$

Conversely, suppose $A \cap B = A$

Let $x \in A$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$

$\therefore A \subset B$

Hence, (i) $\Leftrightarrow$ (iv).

5. Show that if $A \subset B$, then $C - B \subset C - A$.

Ans. Let $A \subset B$

To show: $C - B \subset C - A$

Let $x \in C - B$

$\Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in C$ and $x \notin A$ $[A \subset B]$

$\Rightarrow x \in C - A$

$\therefore C - B \subset C - A$

6. Assume that P (A) = P (B). Show that A = B.

Ans. Let P(A) = P(B)

To show: A = B

Let $x \in A$

$A \in P(A) = P(B)$

$\therefore x \in C$, for some $C \in P(B)$

Now, $C \subset B$

$\therefore x \in B$

$\therefore A \subset B$

Similarly, $B \subset A$

$\therefore A = B$

7. Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Ans. False

Let A = {0, 1} and B = {1, 2}

$\therefore A \cup B = \{0, 1, 2\}$

$P(A) = \{\Phi, \{0\}, \{1\}, \{0, 1\}\}$

$P(B) = \{\Phi, \{1\}, \{2\}, \{1, 2\}\}$

$P(A \cup B) = \{\Phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}, \{0, 2\}, \{0, 1, 2\}\}$

$P(A) \cup P(B) = \{\Phi, \{0\}, \{1\}, \{0, 1\}, \{2\}, \{1, 2\}\}$

$\therefore P(A) \cup P(B) \neq P(A \cup B)$

8. Show that for any sets A and B,

$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = (A \cup B)$

Ans. To show: $A = (A \cap B) \cup (A - B)$

Let $x \in A$

We have to show that $x \in (A \cap B) \cup (A - B)$

Case I $x \in A \cap B$

Then, $x \in (A \cap B) \subset (A \cup B) \cup (A - B)$

Case II $x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\therefore x \notin B$ [$x \notin A$]

$\therefore x \notin A - B \subset (A \cup B) \cup (A - B)$

$\therefore A \subset (A \cap B) \cup (A - B) \qquad \rightarrow(1)$

It is clear that

$A \cap B \subset A$ and $(A - B) \subset A$

$\therefore (A \cap B) \cup (A - B) \subset A \qquad \rightarrow(2)$

From (1) and (2),

$A = (A \cap B) \cup (A - B)$

To prove: $A \cup (B - A) \subset A \cup B$

Let $x \not\subset A \cup (B - A)$

$\Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A$ or ($x \in B$ and $x \notin A$)

$\Rightarrow$ ($x \in A$ or $x \in B$) and ($x \in A$ or $x \notin A$)

$\Rightarrow x \in (A \cup B)$

$\therefore A \cup (B - A) \subset (A \cup B) \qquad \rightarrow(3)$

Next, we show that $(A \cup B) \subset A \cup (B - A)$.

Let $y \in A \cup B$

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y \in A$ or $y \notin A)$

$\Rightarrow y \in A$ or $(y \in B$ and $y \notin A)$

$\Rightarrow y \in A \cup (B - A)$

$\therefore A \cup B \subset A \cup (B - A) \qquad \rightarrow(4)$

From (3) and (4), $A \cup (B - A) = A \cup B$.

9. Using properties of sets show that

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$.

Ans. (i) We know that $A \subset A$

$A \cap B \subset A$

$\therefore A \cup (A \cap B) \subset A \qquad \rightarrow(1)$

Also, $A \subset A \cup (A \cap B) \qquad \rightarrow(2)$

$\therefore$ From (1) and (2), A (A ∩ B) = A

(ii) To show: $A \cap (A \cup B) = A$

$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$

$= A \cup (A \cap B) = A$ {from (1)}

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Ans. Let A = {0, 1}, B = {0, 2, 3}, and C = {0, 4, 5}

Accordingly, $A \cap B = \{0\}$ and $A \cap C = \{0\}$

Here, $A \cap B = A \cap C = \{0\}$

However, $B \neq C \qquad [2 \in B$ and $2 \notin C]$

11. Let A and B be sets. If $A \cap X = B \cap X = \Phi$ and $A \cup X = B \cup X$ for some set X, show that A = B.

(Hints $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use distributive law)

Ans. Let A and B be two sets such that $A \cap X = B \cap X = f$ and $A \cup X = B \cup X$ for some set X.

To show: A = B

It can be seen that

$$A = A \cap (A \cup X) = A \cap (B \cup X) \qquad [A \cup X = B \cup X]$$

$$= (A \cap B) \cup (A \cap X) \qquad [\text{Distributive law}]$$

$= (A \cap B) \cup \Phi$ [$A \cap X = \Phi$]

$= A \cap B$ $\rightarrow(1)$

Now, $B = B \cap (B \cup X)$

$= B \cap (A \cup X)$ [$A \cup X = B \cup X$]

$= (B \cap A) \cup (B \cap X)$ [Distributive law]

$= (B \cap A) \cup \Phi$ [$B \cap X = \Phi$]

$= B \cap A = A \cap B$ $\rightarrow(2)$

Hence, from (1) and (2), we obtain A = B.

12. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \Phi$.

Ans. Let A = {0, 1}, B = {1, 2}, and C = {2, 0}.

Accordingly, $A \cap B = \{1\}$, $B \cap C = \{2\}$, and $A \cap C = \{0\}$.

$\therefore A \cap B$, $B \cap C$, and $A \cap C$ are non-empty.

However, $A \cap B \cap C = \Phi$

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Ans. Given, n(U) = 600, n(T) = 150, n(C) = 225, $n(T \cap C) = 100$

$n(T' \cap C') = n(T \cup C)'$

$= n(U) - n(T \cup C)$

$= n(U) - [n(T) + n(C) - n(T \cap C)]$

$= 600 - [150 + 225 - 100] = 325$

14. In a group of students 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Ans. $H \cup E = U$

Given, $n(H) = 100$ and $n(E) = 50$

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$= 100 + 50 - 25 = 125$

15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I,11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

Ans. Let A be the set of people who read newspaper H. Let B be the set of people who read newspaper T. Let C be the set of people who read newspaper I.

$n(A) = 25, n(B) = 26, n(C) = 26 , n(A \cap C) = 9, n(A \cap B) = 11,$

$n(B \cap C) = 8 , n(A \cap B \cap C) = 3$

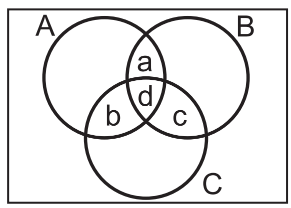
Let U be the set of people who took part in the survey.

(i) $n(A B C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap$

$B \cap C) = 25 + 26 + 26 - 11 - 8 - 9 + 3 = 52$

Hence, 52 people read at least one of the newspapers.

(ii) Let a be the number of people who read newspapers H and T only.



Let b denote the number of people who read newspapers I and H only.

Let c denote the number of people who read newspapers T and I only.

Let d denote the number of people who read all three newspapers.

$d = n(A \cap B \cap C) = 3$

Now, $n(A \cap B) = a + d$

$n(B \cap C) = c + d$

$n(C \cap A) = b + d$

$\therefore a + d + c + d + b + d = 11 + 8 + 9 = 28$

$\Rightarrow a + b + c + d = 28 - 2d = 28 - 6 = 22$

So, $(52 - 22) =$ 30 people read exactly one newspaper.

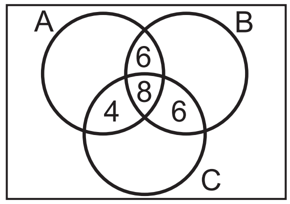
16. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Ans. Let A, B, and C be the set of people who like product A, product B, and product C respectively.

Given, $n(A) = 21, n(B) = 26, n(C) = 29, n(A \cap B) = 14, n(C \cap A) = 12,$

$n(B \cap C) = 14, n(A \cap B \cap C) = 8$

The Venn diagram for the given problem can be drawn as



The number of people who like product C only = {29 – (4 + 8 + 6)} = 11

ADDITIONAL QUESTIONS AND ANSWERS

1. (a) If U is the universal set and A is any set then U ∩ A = ...

(i) U (ii) A (iii) Φ (iv) A'

(b) Consider the sets U = {a,b,c,d,e,f,g}, A = {b,c,d,e} and B = {a,c,g}

Find A' and B' and then verify that (A ∪ B)' = A' ∩ B'.

(c) In a group of 400 people, 250 can speak Hindi and 200 can speak Malayalam. How many people can speak both Hindi and Malayalam?

Ans. (a) (ii) A

(b) U = {a, b, c, d, e, f, g}

A = {b, c, d, e}, B = {a, c, g}

A' = {a, f, g}, B' = {b, d, e, f}

A U B = {b, c, d, e} U {a, c, g}

= {a, b, c, d, e, g},

(A U B)' = { f }

A' ∩ B' = {a, f, g} ∩ {b, d, e, f} = { f }

So (A U B)' = A' ∩ B'

(c) Given, n(H ∪ M) = 400, n(H) = 250, n(M) = 200 n(H ∩ M) = ?

We know that, n(H ∪ M) = n(H) + n(M) – n(H ∩ M)

∴ 400 = 250 + 200 – n(H ∩ M)

⇒ 400 = 450 – n(H ∩ M)

⇒ n(H ∩ M) = 450 – 400 = 50

2. If U = {1,2,3,4,5,6,7,8}, A = {2,4,6,8} and B = {2,4,8}, then:

(a) Write A' and B'

(b) For the above sets A and B, prove that (A∪B)'= A'∩B'

(c) Check whether $(A\cap B)' = A'\cup B'$.

Ans. $U = \{1,2,3, 4,5,6, 7, 8\}$, $A = \{2, 4, 6, 8\}$, $B = \{2,4, 8\}$

(a) $A' = U - A = \{1, 3, 5, 7\}$

$B' = U - B = \{1, 3, 5,6, 7\}$

(b) $A \cup B = \{2,4, 6, 8\} \cup \{2, 4, 8\}$

$= \{2, 4, 6, 8\}$

$(A \cup B)' = U - (A \cup B) = \{1, 3, 5, 7\}$

$A' \cap B' = \{1, 3, 5, 7\} \cap \{1, 3, 5, 6, 7\} = \{1, 3, 5, 7\}$

So $(A \cup B)' = A' \cap B'$

(c) $A \cap B = \{2, 4, 8\}$

$(A \cap B)' = \{1, 3, 5, 6, 7\}$

$A' \cup B' = \{1, 3, 5, 6, 7\}$

So $(A \cap B)' = A' \cup B'$

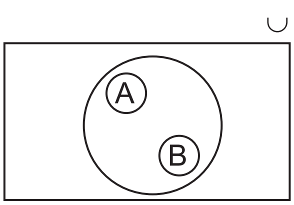
3. (a) If A is a subset of the set B, then $A\cap B$ = ...

(b) Represent the above set $A\cap B$ by Venn diagram.

(c) In a school there are 20 teachers who teach Mathematics or Physics. Of these, 12 teach Mathematics, 12 teach Physics. How many teach both the subjects? **(2016)**

Ans. (a) $A \cap B = A$

(b)



(c) Let the sets defined as follows:

M - Mathematics, P - Physics

$n(m) = 12$, $n(p) = 12$, $n(m \cup p) = 20$

$n(m \cap p) = n(m) + n(p) - n(m \cup p)$

$= 12 + 12 - 20 = 4$

4. (a) A - $\{x / x$, is a prime number, $x \leq 6\}$.

(i) Represent A in roster form.

(ii) Write the power set of A.

(b) Out of 25 members in an office 17 like to take tea, 16 like to take coffee.

Assume that each takes at least one of the two drinks. How many like:

(i) Both coffee and tea? ii) Only tea and not coffee?

Ans. (a) (i) $A = \{2, 3, 5\}$

(ii) $P(A) = \{\phi, \{2\}, \{3\}, \{5\}, \{2, 3\}, \{2, 5\}, \{3, 5\}, \{2, 3, 5\}$

(b) Let the sets be defined as follows.

T - Tea, C- Coffee

$n(T) = 17$, $n(C) = 16$, $n(T \cup C) = 25$

(i) $n(T \cap C) = n(T) + n(C) - n(T \cup C)$

$= 17 + 16 - 25 = 8$

(ii) $n(T \cap C') = n(T) - n(T \cap C)$

$= 17 - 8 = 9$

5. Let $A = \{x : x \in W, x < 5\}$ and

$B = \{x : x$ is a prime number less than 5$\}$, $U = \{x : x$ is an integer, $0 \leq x \leq 6\}$,

(a) Write A, B in roster form. (b) Find $(A - B) \cup (B - A)$

(c) Verify that $(A \cup B)' = A' \cap B'$

Ans. (a) $A = \{0, 1, 2, 3, 4\}$

$B = \{2, 3\}$

(b) $A - B = \{0, 1, 4\}$

$B - A = \{\ \}$

$(A - B) \cup (B - A) = \{0, 1, 4\}$

(c) $\cup = \{0, 1, 2, 3, 4, 5, 6\}$

$(A \cup B)' = \{5, 6\}$

$A' = \{5, 6\}$, $B' = \{0, 1, 4, 5, 6\}$

$A' \cap B' = \{5, 6\}$

So $(A \cup B)' = A' \cap B'$

6. (a) If two sets A and B are disjoint, which one among the following is true?

(i) $A \cup B = A$ (ii) $A \cup B = B$ (iii) $A \cap B = B$ (iv) $A \cap B = \phi$

(b) Find the solution set of the equation : $x^2 + x - 2 = 0$ in roster form.

(c) In a group of students, 100 students know Hindi, 50 know English and 33 know both. Each of the students know either Hindi or English. How many students are there in the group?

Ans. (a) (iv) $A \cap B = \phi$

(b) $x^2 + x - 2 = 0$

$(x - 1)(x + 2) = 0$

$x = 1$ or -2

Solution set is {1, -2}

(c) Let H - Hindi, E - English

n (H) = 100, n (E) = 50, n (H ∩ E) = 33

$n(H \cup E) = n(H) + n(E) + n(H \cap E)$

$= 100 + 50 - 33 = 117$

7. Consider the sets A = {2,3,5,7} and` B = {1,2,3,4,6,12} .

(a) Find A ∩ B

(b) Find A - B, B - A and hence show that (A∩B) ∪ (A - B) ∪ (B - A) = A ∪ B

(c) Write the power set of A ∩ B.

Ans. A = {2, 3, 5, 7}, B = {1, 2, 3,4, 6, 12}

(a) $A \cap B = \{2, 3, 5, 7\} \cap \{1, 2, 3, 4, 6, 12\} = \{2, 3\}$

(b) $A - B = \{5, 7\}$

$B - A = \{1, 4, 6, 12\}$

$(A \cap B) \cup (A - B) \cup (B - A) = \{2, 3\} \cup \{5, 7\} \cup \{1, 4, 6, 12\}$

$= \{1, 2, 3, 4, 5, 6, 7, 12\} = A \cup B$

Hence LHS = RHS

(c) $P(A \cap B) = \{\phi\ \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2,3\}\}$

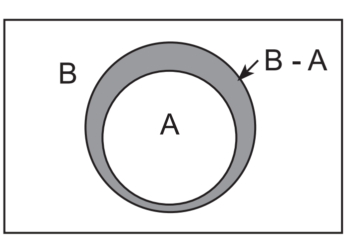
8. If A and B are two sets such that A ⊂ B, A ∪ B is ……………

(a) Draw the Venn diagram of B - A

(b) In a committee, 60 people speak English, 30 speak Hindi and 15 speak both English and Hindi. How many speak atleast one of these two languages?

Ans. A ∪ B = B

(a)



(b) Let the sets defined as E -English, H - Hindi

$n(E) = 60,\ n(H) = 30,\ n(E \cap H) = 15$

$$n(E \cup H) = n(E) + n(H) - n(E \cap H)$$

$$= 60 + 30 - 15 = 75$$

9. Let U = {1,2,3,4,5,6,7,8,9} ; A = {1,2,4,7} and B = {1,3,5,7}

(a) Find $A \cup B$

(b) Find A' , B' and hence show that $(A \cup B)' = A' \cap B'$.

(c) The power set $P(A \cup B)$ has………elements.

Ans. (a) $A \cup B = \{1, 2, 4, 7\} \cup \{1, 3, 5, 7\}$

$= \{1, 2, 3, 4, 5, 7\}$

(b) $A' = \cup - A = \{3, 5, 6, 8, 9\}$

$B' = \cup - B = \{2, 4, 6, 8, 9\}$

$(A \cup B)' = \cup - (A \cup B) = \{6, 8, 9\}$

$A' \cap B' = \{6, 8, 9\}$

So $(A \cup B)' = A' \cap B'$

(c) $2^4 = 16$

10. (i) How many elements have P(A), if A = {1,2,3}?

(ii) U = {1,2,3,4,5,6,7}; A = {1,4,6,7}; and B = {1,2,3}. Find A', B', $A' \cap B'$, $A \cup B$. Hence show that $(A \cup B)' = A' \cap B'$

(iii) If A and B are two sets such that $A \subset B$ then what is $A \cap B$?

Ans. (i) $n(P(A)) = 2^3 = 8$

(ii) $A' = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 4, 6, 7\}$

$= \{2, 3, 5\}$

$B' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3\}$

$= \{4, 5, 6, 7\}$

$A' \cap B' = \{2, 3, 5\} \cap \{4, 5, 6, 7\} = \{5\}$

$A \cup B = \{1, 4, 6, 7\} \cup \{1, 2, 3\} = \{1, 2, 3, 4, 5, 6, 7\}$

$(A \cup B)' = U - A \cup B$

$= \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4, 6, 7\} = \{5\}$

Hence $A' \cap B' = (A \cup B)'$

(iii) $A \cap B = A$

11. Let A = {x : x is an integer, $\frac{1}{2} < x < \frac{7}{2}$},

B = {2,3,4}

(a) Write A in the roster form. (b) Find the power set of A ∪ B

(c) Verify that (A -B) ∪ (A ∩ B) = A

Ans. (a) A = {1, 2, 3}

B = {2, 3, 4}

(A ∪ B) = {1, 2, 3, 4}

(b) p(A ∪ B) = {ϕ, {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4},

{2, 2}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4},

{1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}}

(c) A - B = {1, 2, 3} - {2, 3, 4} = {1}

A ∩ B = {1, 2, 3} ∩ {2, 3, 4} = {2, 3}

(A - B) ∪ (A ∩ B) = {1} ∪ {2, 3} = {1, 2, 3} = A

12. Let U = {x : x is an integer, $-4 < x < 4$} be the universal set.

A = {x : x is a integer ,$0 \le x \le 3$] and

B = {x : x is a integer ,$-3 < x < 1$} are the subsets of U.

(a) Verify A in the roster form. (b) Verify (A ∪ B)' = A' ∩ B'

(c) Write the power set of A ∩ B

Ans. U = {-3, -2, -1, 0, 1, 2, 3}

(a) A = {0, 1, 2, 3}

(b) B = {-2, -1, 0}

(A ∪ B) = {0, 1, 2, 3} ∪ {-2, -1, 0}

= {-2, -1, 0, 1, 2,3}

(A ∪ B)' = U - A ∪ B

= {-3, -2, -1, 0, 1, 2, 3} - {-2, -1, 0, 1, 2, 3} = {-3}

A' = ∪ - A = {-3, -2, -1}

B' = ∪ - B = {-3, 1, 2, 3} = {-3}

A' ∩ B' = {-3, -2, -1} ∩ {-3, 1, 2, 3} = {-3}

So (A ∪ B)' = A' ∩ B'

(c) A ∩ B = {0, 1, 2, 3} ∩ {-2, -1, 0} = {0}

$P(A \cap B) = \{\phi, \{0\}\}$

ENTRANCE CORNER

1. In a certain town, 25**%** of the families own a phone and 15% own a car, 65% families own neither a phone nor a car and 2000 families own both a car and a phone. Consider the following three statements:

   (1) 5% own both a car and a phone.

   (2) 35% families own either a car or a phone.

   (3) 40000 families live in the town. Then

   (a) Only (1) and (2) are correct. (b) Only (1) and (3) are correct.
   (c) Only (2) and (3) are correct. (d) All (1), (2) and (3) are correct. **(2015)**

2. Let X = {1,2,3,4,5}. The number of different ordered pairs (Y,Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is

   (a) 2^5 (b) 5^3 (c) 5^2 (d) 3^5 **(2012)**

3. The set S = {1,2,3,........} is to be partitioned in to three sets A, B, C of equal size. Thus $A \cup B\ B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

   (a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$ (c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$

4. If f: $R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

   (a) [o, 1] (b) [-1, 1] (c) [0, 3] (d) [-1, 3] **(2004)**

5. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

   (a) $f(x) = f(-x)$ (b) $f(2 + x) = f(2 - x)$ **(2004)**

   (c) $f(x + 2) = f(x - 2)$ (d) $f(x) = -f(-x)$

6. The period of $\sin^2\theta$ is: (a) π^2 (b) π (c) π^3 (d) $\pi/2$.

Answers

1 (d) 2 (d) 3 (a) 4 (d) 5 (c) 6 (b)

✧✧✧✧✧