TEACHERS FORUM™

QUESTION BANK

(solved)

Class XI

PHYSICS

SUBJECT EXPERTS

CONTENTS

1 UNITS AND MEASUREMENTS

THE INTERNATIONAL SYSTEM OF UNITS

• In CGS system they were centimetre, gram and second respectively.

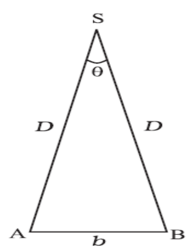
• In FPS system they were foot, pound and second respectively.

• In MKS system they were metre, kilogram and second respectively.

The system of units which is at present internationally accepted for measurement is the Système Internationale d' Unites, abbreviated as SI. In SI, there are seven base units.

Measurement of Large Distances

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method.



ACCURACY, PRECISION OF INSTRUMENTS AND ERRORS IN MEASUREMENT

The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty is called error.

The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured. The errors in measurement can be broadly classified as

(a) systematic errors and (b) random errors.

Systematic errors

The systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are :

(a) Instrumental errors that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(b) Imperfection in experimental technique or procedure.

(c) Personal errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc. Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.

Random errors

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions, personal errors by the observer taking readings, etc.

Least count error

The smallest value that can be measured by the measuring instrument is called its least count. The least count error is the error associated with the resolution of the instrument.

Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error. Repeating the observations several times and taking the arithmetic mean of all the observations, the mean value would be very close to the true value of the measured quantity.

NCERT SOLUTIONS

1. Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to.....m^3

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ... $(mm)^2$

(c) A vehicle moving with a speed of 18 km h^{-1} covers....m in 1 s

(d) The relative density of lead is 11.3. Its density is ....g cm^{-3} or . ...kg m^{-3}.

Ans. (a) $1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm = \left(\frac{1}{100}\right) m \times \left(\frac{1}{100}\right) m \times \left(\frac{1}{100}\right) m = 10^{-6}\ m^3$

(b) T.S.A $= 2\pi r\,(r + h)$.

$$= 2 \times \frac{22}{7} \times 20 \times (20+100) = 1.5 \times 10^4\ mm^2$$

(c) $18 km/h = 18 \times \frac{5}{18} = 5\ m/s$

So, the vehicle covers 5 m in 1 s.

(d) Relative density $= \dfrac{\text{Density of substance}}{\text{Density of water}}$

Density of lead = relative density of lead x density of water

$$= 11.3 \times 1 = 11.3\ g/cm^3$$

Now, $1g = \frac{1}{1000}\ kg = 10^{-3}\ kg$

$1\ cm^3 = 10^{-6}\ m^3$

$1\ g/cm^3 = \frac{10^{-3}}{10^{-6}} = 10^3\ kg/m^3$

$\therefore$ 11.3 g/cm^3 = 11.3 × 10^3 kg/m^3

2. Fill in the blanks by suitable conversion of units:

(a) 1 kg m^2s^{-2} = ...g cm^2 s^{-2} (b) 1 m =..... ly

(c) 3.0 m s^{-2}=.... km h^{-2} (d) G = 6.67 × 10^{-11} N m^2 $(kg)^{-2}$ =.... $(cm)^3$ s^{-2} g^{-1}

Ans. (a) 10^7 ; (b) 10^{-16} ; (c) 3.9 × 10^4 ; (d) 6.67 × 10^{-8}.

3. A calorie is a unit of heat or energy and it equals about 4.2 J where 1J = 1 kg m^2s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 $\alpha^{-1}\beta^{-2}\gamma^2$ in terms of the new units.

Ans. Given,

SI	New system
n_1 = 4.2	n_2 = ?
m_1 = 1 kg	m_2 = α kg
L_1 = 1m	L_2 = β m
T_1 = 1s	T_2 = y s

For energy, dimensional formula is $M^1 L^2 T^{-2}$

Comparing with [$M^a L^b T^c$], we get a = 1, b = 2, and c = -2

$$\therefore\ n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2 \left[\frac{1\ kg}{\alpha K}\right]^1 \left[\frac{1m}{\beta m}\right]^2 \left[\frac{1s}{\gamma s}\right]^{-2}$$

ie., $n_2 = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$

5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Ans. Distance between the Sun and the Earth

= Speed of light × Time taken by light to cover the distance

= 1 unit × 500 s

= 1 × 500 = 500 units

6. Which of the following is the most precise device for measuring length:

(a) a vernier callipers with 20 divisions on the sliding scale
(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
(c) an optical instrument that can measure length to within a wavelength of light ?

Ans. (a) Least count of vernier callipers = 1 standard division (SD) – 1 vernier division (VD)

$$= 1 - \frac{9}{10} = \frac{1}{10} = 0.01 \text{ cm} = 1 \times 10^{-4} \text{ m}$$

(b) Least count of screw gauge $= \frac{\text{Pitch}}{\text{Number of Divisions}}$

$$= \frac{1 \times 10^{-3}}{100} = 1 \times 10^{-5} \text{ m}$$

(c) Least count of an optical device = Wavelength of light = 10^{-5} cm = 1×10^{-7} m
So, an optical instrument is the most precise device for measuring length.

7. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the thickness of hair?

Ans. Given, magnification = 100

Average width of the hair = 3.5 mm

$\therefore$ Thickness of the hair = $\frac{3.5}{100}$ = 0.035 mm.

9. The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector?

Ans. Area of the house on the slide = 1.75 cm^2

Area of the image of the house on the screen = 1.55 m^2 = 1.55×10^4 cm^2

$$\text{Areal magnification} = \frac{\text{Area of image}}{\text{Area of object}} = \frac{1.55}{1.75} \times 10^4 = 0.8857 \times 10^4$$

$$\therefore \text{Linear magnifications} = \sqrt{\text{Areal magnification}} = \sqrt{0.8857 \times 10^4} = 94.1$$

10. State the number of significant figures in the following:

(a) 0.007 m^2 (b) 2.64×10^{24} kg (c) 0.2370 g cm^{-3}

(d) 6.320 J (e) 6.032 N m^{-2} (f) 0.0006032 m^2

Ans. (a) 1 ; (b) 3 ; (c) 4 ; (d) 4 ; (e) 4 ; (f) 4.

11. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Ans. Given, l = 4.234 m ; significant figures = 4

b = 1.005 m ; significant figures = 4

h = 2.01 cm = 0.0201 m ; significant figures = 3

Surface area of the sheet $= 2(l \times b + b \times h + h \times l)$

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)$

$= 2(4.25517 + 0.02620 + 0.08510) = 2 \times 4.360$

$= 8.72$ m^2

Volume of the sheet = $l \times b \times h$ = $4.234 \times 1.005 \times 0.0201 = 0.0855\ m^3$

12. The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

Ans. (a) Total mass of the box = 2.3 + 0.02015 + 0.02017 = 2.34032 kg
Since the least number of decimal places is 1, the total mass of box = 2.3 kg

(b) Difference in masses = 20.17 - 20.15 = 0.028

Since the least number of decimal places is 2, the correct significant figure is 0.02 g

13. A physical quantity P is related to four observables *a*, *b*, *c* and *d* as : $P = \frac{a^3b^2}{(\sqrt{cd})}$

The percentage errors of measurement in *a*, *b*, *c* and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity *p?*

If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Ans.

$$P = \frac{a^3b^2}{(\sqrt{cd})}$$

$$\frac{\Delta P}{P} = \frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c}$$

$$\left(\frac{\Delta P}{P} \times 100\right)\% = \left(3 \times \frac{\Delta a}{a} \times 100 + 2 \times \frac{\Delta b}{b} \times 100 + \frac{1}{2} \times \frac{\Delta c}{c} \times 100 + \frac{\Delta d}{d} \times 100\right)\%$$

$$= 3 \times 1 + 2 \times 3 + \frac{1}{2} \times 4 + 2$$

$$= 3 + 6 + 2 + 2 = 13\%$$

Percentage error in *P* = 13 %.

Value of P is given as 3.763. By rounding off the given value, we get P = 3.8

14. A book with many printing errors contains four different formulas for the displacement of a particle undergoing a certain periodic motion:

(a) $y = a \sin \frac{2\pi t}{T}$ (b) $y = a \sin vt$

(c) $y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$ (d) $y = \left(a\sqrt{2}\right)\left[\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right]$

(*a* = maximum displacement of the particle, *v* = speed of the particle. *T* = time-period of motion). Rule out the wrong formulas on dimensional grounds.

Ans. (a) $y = \sin \frac{2\pi t}{T}$

LHS = [Y] = L

RHS = $[\ a \sin \frac{2\pi t}{T}\ ] = L$

So, the given formula is dimensioually correct

(b) $y = a \sin vt$

LHS = [Y] = L

RHS = [a sin vt] = [L × sin LT^{-1} T] = [L sin L]

So, the given formula is dimensionally incorrect.

(c) $y = \frac{a}{T} \sin \frac{t}{a}$

LHS = [Y] = L

RHS = $\left[\frac{a}{T} \sin \frac{t}{a} \right]$ = [LT^{-1} sin TL^{-1}]

So, the formula is dimensionally incorrect.

(d) $y = \left(a\sqrt{2}\right) \left[\sin 2\pi \frac{t}{T} + \cos 2\pi \frac{t}{T}\right]$

LHS = [Y] = L

RHS = $[a\sqrt{2}]$ = [L]

and $\left[\sin 2\pi \frac{t}{T} + \cos 2\pi \frac{t}{T}\right] = \left[\sin \frac{T}{T} + \cos \frac{T}{T}\right]$ = dimensionless

So the given formula is dimensionally correct.

15. A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light,c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes: $m = \frac{m_0}{(1 - v^2)^{½}}$

Ans. Given the relation can be written as $m = \frac{m_o}{\sqrt{1- v^2}}$

ie., $\frac{m_0}{m} = \sqrt{1 - v^2}$

LHS = dimensionless.

So the formula will be correct if $\sqrt{(1 - v^2)}$ is dimensionless. This is only possible if V^2 is divided by c^2 . So the correct relation is

$$m = \frac{m_o}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

16. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by **A** : 1**A** = 10^{-10} m. The size of a hydrogen atom is about 0.5 **A**. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Ans. Volume of hydrogen atom $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3 = 5.23 \times 10^{-31}\ m^3$

1 mole of hydrogen contains 6.023×10^{23} hydrogen atoms.

$\therefore$ Atomic volume of 1 mole of hydrogen atoms = $6.023 \times 10^{23} \times 5.23 \times 10^{-31}$

$= 3.15 \times 10^{-7}\ m^3$

17. One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1**A**). Why is this ratio so large?

Ans. Volume of one mole of ideal gas, $V_g = 22.4 \times 10^{-3}\ m^3$.

Volume of hydrogen atom $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3 = 0.524 \times 10^{-30}\ m^3$

1 mole contains 6.023×10^{23} molecules.

$\therefore$ Volume of 1 mole of hydrogen, $V_H = 6.023 \times 10^{23} \times 0.524 \times 10^{-30}$

$= 3.16 \times 10^{-7}\ m^3$

$$\therefore \frac{V_g}{V_H} = \frac{22.4 \times 10^{-3}}{3.16 \times 10^{-7}} = 7.08 \times 10^4$$

The ratio is very large, because the inter-atomic separation in hydrogen gas is much larger than the size of a hydrogen molecule.

18. Explain this common observation clearly : If you look out of the window of a fast mooving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

Ans. Near objects make greater angle than distant (far off) objects at the eye of the observer. When you are moving, the angular change is less for distant objects than nearer objects. So, these distant objects seem to move along with you, but the nearer objects in opposite direction.

19. The principle of 'parallax' in section 2.3.1 is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit $\approx 3 \times 10^{11}$m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres ?

Ans. Given, $b = 3 \times 10^{11}$ m, $r = 1.5 \times 10^{11}$ m, $\theta = 1'' = 4.847 \times 10^{-6}$ rad

Let the distance of the star be D

From parallax method, $\theta = \frac{r}{D}$

$\Rightarrow D = \frac{r}{\theta} = \frac{1.5 \times 10^{11}}{4.847 \times 10^{-6}}$

$= 3.09 \times 10^{16}$ m $\approx 3 \times 10^{16}$ m

20. The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named *Alpha Centauri)* show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

Ans. Distance of the star from the solar system = 4.29 ly

1 light year is the distance travelled by light in one year = 9.46×10^{15} m

$\therefore 4.29$ ly = 4.058×10^{16} m

1 parsec = 3.08×10^{16} m

$\therefore 4.29 \text{ ly} = \frac{4058 \times 10^{16}}{3.08 \times 10^{16}} = 1.32$ parsec

Now parallax angle subtended by 1 parsec distance = 2 second. (by definition)

$\therefore$ Parallax angle subtended by the star, $\theta = 1.32 \times 2 = 2.64''$

23. The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures,no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.

Ans. Given, $M = 2.0 \times 10^{30}$ kg, $R = 7.0 \times 10^8$ m

$\therefore$ Volume of the Sun, $V = \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \frac{22}{7} \times (7.0 \times 10^8)^3 = 1437 \times 10^{27}\ m^3$

Now density of the Sun $= \frac{\text{Mass}}{\text{Volume}} = \frac{2.0 \times 10^{30}}{1437 \times 10^{27}} = 1.4 \times 10^3\ kg/m^{-3}$

The mass density of the Sun is in the range of densities of liquids /solids and not gases. This high density arises due to inward gravitational attraction on outer layers due to inner layers of the Sun.

24. When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72" of arc. Calculate the diameter of Jupiter.

Ans. Given, Distance of Jupiter from the Earth, $d = 824.7 \times 10^6$ km = 824.7×10^9 m

Angular diameter, $\theta = 35.72'' = 35.72 \times 4.85 \times 10^{-6}$ rad

Diameter of Jupiter, $D = \theta \times d = 35.72 \times 4.85 \times 10^{-6} \times (824.7 \times 10^9) = 1.43 \times 10^8$ m

25. A man walking briskly in rain with speed v must slant his umbrella forward making an

angle θ with the vertical. A student derives the following relation between θ and v: tan θ = v and checks that the relation has a correct limit: as v →0, θ → 0, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Ans. Given, $\tan\theta = v$.

Dimension of R.H.S = $M^0 L^1 T^1$

Dimension of L.H.S = $M^0 L^0 T^0$

(The trigonometric function is considered as a dimensionless quantity)

Dimension of R.H.S is not equal to the dimension of L.H.S.

So to make the given relation correct, the R.H.S also should be dimensionless.

On correcting LHS we get $\tan\theta = \frac{v}{u}$, where u is velocity of rain.

26. It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?

Ans. Total time = 100 years = $100 \times 365 \times 24 \times 60 \times 60 = 3.15 \times 10^9$ s

In 3.15×10^9 s, the caesium clock shows a time difference of 0.02 s.

$\therefore$ Error In 1s = $\frac{0.02}{3.15 \times 10^9}$ s = 6.34×10^{-12} s

$\Rightarrow$ Accuracy of 1 part in 10^{11} to 10^{12}

27. Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: 970 kg m^{-3} Are the two densities of the same order of magnitude? If so, why?

Ans. Given, diameter of sodium atom = 2.5 Å

$\therefore$ Radius of sodium atom, $r = \frac{1}{2} \times 2.5$ Å = 1.25 Å= 1.25×10^{-10} m

Volume of one mole of sodium atom, $V = N_A \frac{4}{3} \pi r^3$

$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (1.25 \times 10^{-10})^3$

Mass of one mole of sodium atom = 23×10^{-3} kg

$\therefore$ Mass of one atom = $\frac{23 \times 10^{-3}}{6.023 \times 10^{23}}$ kg

$\therefore$ Average density of sodium atom = $\frac{23 \times 10^{-3}}{6.023 \times 10^{23} \times 4/3 \times 3.14 \times (2.5 \times 10^{-10})^2}$

$= 0.7 \times 10^{-3}$ kgm^{-3}

It is given that the density of sodium in crystalline phase is 970 kg m^{-3} . So, the density of sodium atom and the density of sodium in its crystalline phase are not in the same order. This is because In the solid phase atoms are tightly packed, so the atomic massdensity is close to the mass density of the solid.

29. A LASER is a source of very intense, monochromatic, and unidirectional beam of light These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Ans. Time taken by the laser beam to reach Moon = $\frac{1}{2}$ x 2.56 = 1.28 s

$\therefore$ Radius of the lunar orbit = Distance between the Earth and the Moon

= time × speed = 1.28 × 3 × 10^8 = 3.84 × 10^8 m

30. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).

Ans. Time taken for the sound to reach the submarine = $\frac{1}{2}$ x 77 = 38.5 s

$\therefore$ Distance between the ship and the submarine = speed × time

= 1450 × 38.5 = 55825 m = 55.8 km

31. The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Ans. Time taken by quasar light to reach Earth = 3 billion years = 3 × 10^9 years

= 3 × 10^9 × 365 × 24 × 60 × 60 s

Speed of light = 3 × 10^8 m/s

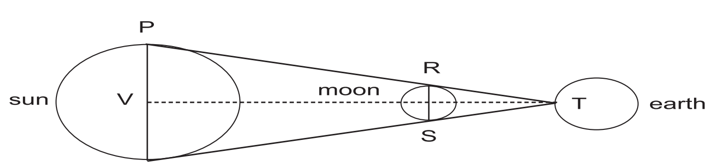
Distance between the Earth and quasar = speed × time

= (3 × 10^8) × (3 × 10^9 × 365 × 24 × 60 × 60)

= 283824 × 10^{20} m = 2.8 × 10^{22} km

32. It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.

Ans. The position of the Sun, Moon, and Earth during a lunar eclipse is shown in the given figure.



Distance of the Moon from the Earth = 3.84×10^8 m

Distance of the Sun from the Earth = 1.496×10^{11}m

Diameter of the Sun = 1.39×10^9 m

It can be observed that ΔTRS and ΔTPQ are similar. Hence, it can be written as:

$$\frac{PQ}{RS} = \frac{VT}{UT}$$

$$\frac{1.39 \times 10^9}{RS} = \frac{1.496 \times 10^{11}}{3.84 \times 10^8}$$

$$\Rightarrow \quad RS = \frac{1.39 \times 3.84}{1.496} \times 10^6 = 3.57 \times 10^6 \text{ m}$$

Hence, the diameter of the Moon is 3.57×10^6 m.

ADDITIONAL QUESTIONS AND ANSWERS

1. (a) Length of a sheet is 17.3 ± 0.3cm and breadth is 3.12 ± 0.08cm. Calculate the percentage error in the area.

   (b) Using the principle of homogeneity of equations, check whether the equation is correct. $T = 2\pi\sqrt{\frac{g}{l}}$

   T → time, g → acceleration l → length

Ans. (a) Area, a $= l \times b$

$$\% \text{ error in a} = \frac{\Delta l}{l} \times 100 + \frac{\Delta b}{b} \times 100$$

$$= \frac{0.3}{17.3} \times 100 + \frac{0.08}{3.12} \times 100$$

$$= 1.73 + 2.56 = 4.29 \%$$

(b) $T = 2\pi\sqrt{\frac{g}{l}}$

Squaring on both sides, $T^2 = 4\pi^2 \frac{g}{l}$

LHS = T^2 = [T^2]

RHS = [LT^{-2}] [L] = [$L^2 T^{-2}$]

LHS ≠ RHS, so the equation is incorrect.

2. (a) The error in the measurement of radius of a circle is 0.6 %. Find the percentage error in the calculation of the area of the circle.

   (b) Name the principle used to check the correctness of an equation.

   (c) What is the number of significant figures in 0.00820 J ?

Ans. (a) Given $\frac{\Delta r}{r} = 0.6 \%$

Area of circle a = πr^2

$\therefore$ percentage error, $\frac{\Delta a}{a} = \frac{2\Delta r}{r} = 2 \times 0.6 = 1.2$ %

(b) The principle of homogeneity.

(c) 3

3. The correctness of equations can be checked using the principle of homogeneity.

(a) State the princple of homogeneity.

(b) Using this principle, check whether the following equation is dimensionally correct.

$\frac{1}{2} mv^2 = mgh$, where m is the mass of the body, v is its velocity, g is the acceleration due to gravity and h is the height.

(c) If percentage errors of measurement in velocity and mass are 2% and 4% respectively, what is the percentage error in kinetic energy ?

Ans. (a) According to this principle, the dimensions of the fundamental quantities are the same in each term on either side of the equation.

Consider the relation a = b + c, where a, b and c are physical quantities. If this equation is correct then dimension of 'a' = dimension of 'b' = dimension of 'c'. This is known as principle of homogeneity of dimensions.

(b) Dimension of $(\frac{1}{2} MV^2) = M \times (LT^{-1})^2 = ML^2T^{-2}$

Dimension of (mgh) $= M \times (LT^{-2}) \times L = ML^2T^{-2}$

The dimension is same on either side of the equation. Hence equation is dimensionally correct.

(c) $KE = \frac{1}{2} mv^2$ given, $\frac{\Delta m}{m} = 2\%$, $\frac{\Delta V}{V} = 4\%$

$\therefore \frac{\Delta KE}{KE} = \frac{\Delta m}{m} + \frac{2\Delta V}{V} = 2\% + 2 \times 4\% = 10\%$

4. (a) Suggest a method to measure the diameter of the moon.

(b) Length, Breadth and thickness of a block is measured using vernier calipers.The percentage errors in the measurements are 2%, 1% and 3 % respectively. Estimate the percentage error in its volume.

(c) A physical quantity is given by $h = \frac{Fv^2}{L}$. F is the force, v is the velocity and L is the angular momentum. Find the dimensions of h.

Ans. (a) Parallax method.

(b) We know that volume, $V = l \times b \times h$

Now percentage error in volume, $\frac{\Delta V}{V} \times 100 = \frac{\Delta l}{l} \times 100 + \frac{\Delta b}{b} \times 100 + \frac{\Delta h}{h} \times 100$

$= 2\% + 1\% + 3\% = 6\%$

(c) Dimensional formula of $F = mLT^{-2}$

Dimensional formula of V = LT^{-1}

Dimensional formula of L = ML^2T^{-1}

$$\therefore h = \frac{MLT^{-2} \times (LT^{-1})^2}{ML^2T^{-1}} = LT^{-3}$$

5. Dimensional method helps in converting the units from one system into another.

(a) Name the principle used for the above purpose.

(b) Using dimension, prove 1 Newton = 10^5 dynes.

Ans. (a) The principle of homogenity.

(b) Newton is unit of force in SI system and dyne is that in the C.G.S system.

Force = mass × acceleration

[Force] = MLT^{-2}

From the dimensional formula of force we have

1 Newton = 1 kg × 1 m × $(1s)^{-2}$ = 1000 gm × 100 cm × $(1s)^{-2}$

= 10^5 × 1 gm × cm × $1s^{-2}$

By definition,

1 dyne = 1 gm × 1 cm × $1s^{-2}$

$\therefore$ 1 Newton = 10^5 dynes

6. Significant figures determine the accuracy of the measurement of a physical quantity.

(a) The radius of a sphere is given by R = 1.03 m. How many significant figures are there in it?

(b) If the percentage error in calculating the radius of the sphere is 2%, what will be the percentage error in calculating the volume?

Ans. (a) 3

(b) Volume of sphere, $V = \frac{4}{3}\pi r^3$

Percentage error in volume, $\frac{\Delta v}{v} \times 100 = 3\ \frac{\Delta r}{r} \times 100$

$= 3 \times 2 = 6\%$

7. Velocity of sound depends on density (ρ) and modulus of elasticity (E). (The dimensional formula of E is $ML^{-1}T^{-2}$).

(a) State the principle of homogeneity.

(b) Using the above principle, arrive at an expression for the velocity of sound. (Take K = 1)

Ans. (a) According to this principle, the dimensions of the fundamental quantities are the same in each term on either side of the equation.

(b) Given, $V \alpha \rho^a \; E^b$

$V = K \rho^a \; E^b$

ie, $V = \rho^a \; E^b$ .... (1) [since K = 1]

Writing the dimensional formula on both sides, we get

$LT^{-1} = [M L^{-3}]^a [M L^{-1} T^{-2}]^b$ [since Density = ML^{-3}]

$M^0L^1T^{-1} = M^{a+b} L^{-3a-b} T^{-2b}$

Comparing the dimensions on both the sides, we get

$a + b = 0$ .... (2)

Also $-2b = -1$

$\Rightarrow b = \frac{1}{2}$

Now (2) $\Rightarrow a + \frac{1}{2} = 0$

$\Rightarrow a = \frac{-1}{2}$

Putting values of a and b in equation (1), we get

$V = \rho^{-1/2} \; E^{1/2}$

$V = \sqrt{\frac{E}{\rho}}$

8. (a) A boy recalls the relativistic mass wrongly as

$$m = \frac{m_0}{\sqrt{1 - v^2}}$$

Using dimensional method put the missing 'c' at proper place.

(b) Name and state the principle used in solving the above problem.

Ans. (a) By the principle of homogenity, dimension [m] should be same as dimension of [m_0]

For this $\sqrt{1 - v^2}$ becomes dimensionless when we put C as $\sqrt{1 - v^2/c^2}$

(b) Principle of homogeneity of dimensions:- According to this principle, the dimensions of the fundamental quantities are the same in each term on either side of the equation.

9. All physical quantities can be expressed in terms of dimension.

(a) Write the physical quantities of the following dimensions:

(i) $[M^1 L^1 T^{-1}]$ (ii) $[M^2 L^2 T^{-2}]$

(b) Check whether the equation

$T = 2\pi\sqrt{\frac{m}{g}}$ is dimensionally correct.

T → Time period of a simple pendulum

m → mass of the bob,   g → acceleration due to gravity

Ans. (a) (i) Momentum   (ii) Work energy

(b) $T = 2\pi\sqrt{\frac{m}{g}}$

Squaring on both sides, we get

$T^2 = 4\pi^2 \frac{m}{g}$

Taking dimensions on both sides,

$T^2 = \frac{M^1}{L^1 T^{-2}}$ $\Rightarrow T^2 = M^1 L^{-1} T^2$

L.H.S ≠ R.H.S

So the equation is incorrect.

10. (a) A student was asked to write the equation of displacement at any instant in a simple harmonic motion of amplitude 'a'. He wrote the equation as $y = a \sin\frac{2\pi v}{k} t$. where 'v' is the velocity at instant 't'. For the equation to be dimensionally correct, what should be the dimensions of k?

(b) What is the area of a square of side 1.4 cm in proper significant figures?

Ans. (a) Here dimension of y = L,

Here dimension of a = L

Here dimension of v = LT^{-1}

Here dimension of t = T

For LHS = RHS, dimension of k = L

(b) Area = $a^2 = 1.4^2 = 1.96\ cm^2 = 2\ cm^2$

11. Give examples for the following:

(a) A dimensionless, unitless physical quantity.

(b) A dimensionless physical quantity but having unit in SI system.

(c) Two physical quantities which have the same dimensions.

Ans. (a) Relative density

(b) Solid angle

(c) Work and energy

12. Mechanical power is represented by $P = Fv + Av^3p$ where, F is the force, v is the velocity, A is the area and ρ is the density.

(a) The dimensional formula of power is ....

(b) Check the dimensional validity of the above equation.

(c) Which of the following equations can't be obtained by the dimensional method?

(i) $T = K\sqrt{\frac{l}{g}}$ (ii) $E = km^2$ (iii) P = h p g (iv) $N = N_o e^{-\lambda t}$

Ans. (a) Power = Work / Time = $\frac{FS}{T}$

$$= \frac{ML^2 T^{-2}}{T} = ML^2 T^{-3}$$

(b) $P = Fv + Av^3\rho$

$ML^2T^{-3} = mav + Av^3\rho$

$$= M \times \frac{L}{T^2} \times \frac{L}{T} + L^2 (\frac{L}{T})^3 \frac{M}{L^3}$$

$ML^2 T^{-3} = ML^2 T^{-3} + ML^2 T^{-3}$

ie. the equation is correct.

(c) (iv) $N = N_o e^{-\lambda t}$

13. Match the following:

a	Coefficient of viscosity	Force/area / a number	$ML^{-1}T^{-2}$
b	Gravitational constant	Force/area / Velocity gradient	$M^{-1}L^3T^{-2}$
c	Modulus of elasticity	Force ×(distance)^2 / (mass)^2	$ML^{-1}T^{-1}$

Ans.

a	Coefficient of viscosity	Force/area / Velocity gradient	$ML^{-1}T^{-1}$
b	Gravitational constant	Force ×(distance)^2 / (mass)^2	$M^{-1}L^3T^{-2}$
c	Modulus of elasticity	Force/area / a number	$ML^{-1}T^{-2}$

14. Consider an equation $\frac{1}{2}$ m v^2 = m g h, where m is the mass of the body, v its velocity, g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct.

Ans. The dimensions of LHS are $[M] [L T^{-1}]^2 = [M] [L^2 T^{-2}]$

$$= [M L^2 T^{-2}]$$

The dimensions of RHS are $[M][L T^{-2}] [L] = [M][L^2 T^{-2}]$

$$= [M L^2 T^{-2}]$$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

15. What are the uses of dimensional equations?

Ans. (i) To check the correctness of the given equation.

(ii) Conversion of one system of units into another.

(iii) To find the dimension of constants in an equation.

16. The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is 1° 54′ . Given the diameter of the Earth to be about 1.276×10^7 m, compute the distance of the moon from the Earth.

Ans. We have $\theta = 1° 54' = 114'$

$= (114 \times 60)'' \times (4.85 \times 10^{-6})$ rad

$= 3.32 \times - 10^{-2}$ rad, [since 1” = 4.85×10^{-6}rad.]

Also $b = AB = 1.276 \times 10^7$m

the earth-moon distance, $D = b/\theta$

$$= \frac{1.276 \text{ x } 10^7}{3.32 \text{ x } 10^{-2}} = 3.84 \times 10^8 \text{m}$$

ENTRANCE CORNER

1. If the displacement of a body varies as the square of elapsed time, then its

(a) velocity is constant (b) velocity varies non-uniformly
(c) acceleration is constant (d) acceleration changes continuously
(e) momentum is constant **(2015)**

2. The time required to stop a car of mass 800 Kg, moving at a speed of $20ms^{-1}$ over a distance of 25m is

(a) 2 s (b) 2.5 s (c) 4 s (d) 4.5 s (e) 1 s **(2015)**

3. Two cars started moving with initial velocities v and 2v. For the same deceleration, their respective stopping distances are in the ratio

(a) 1:1 (b) 1:2 (c) 1:4 (d) 2:1 (e) 4:1 **(2012)**

4. A car move a distance of 200 Km. It covers first half of the distance at aspeed 60 Km h^{-1} and a second half at speed v. Ifthe average speed is 40 Km h^{-1}, the value of v is

(a) 30 Km h^{-1} (b) 13 Km h^{-1} (c) 60 Km h^{-1} (d) 40 km h^{-1} (e) 20 Km h^{-1} **(2011)**

Ans. 1. (c) 2. (b) 3. (c) 4. (a)

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