1 ARITHMETICAL SEQUENCES

TEXT BOOK QUESTIONS & ANSWERS

Page : 10

1. Make the following number sequences, from the sequence of equilateral triangles, squares, regular pentagons and so on, of regular polygons:

   Number of sides : 3, 4, 5, ……

   Sum of inner angles :

   Sum of outer angles :

   One inner angle :

   One outer angle :

Ans. Sum of the interior angles of a polygon with n sides = (n − 2) × 180°

Sum of interior angles of a triangle is 180°.

Then the sequence of sum of interior angles = 180, 180 × 2, 180 × 3 ….

= 180, 360, 540, 720 …….

Sum of exterior angles of any number of sides is always 360°.

So the sequence is: 360^0, 360^0, ……

One interior angle = $\frac{180}{3}, \frac{360}{4}, \frac{540}{5}$, ...

So the sequence is: 120^0, 90^0, 108^0, ……

One outer angle = 180 - 60, 180 - 90, 180 - 108, ...

So the sequence is: 120^0, 90^0, 72^0, …….

2. Look at these triangles made with dots. How many dots are there in each ?

Compute the number of dots needed to make the next three triangles.

Ans. 3, 6, 10

The number of dots needed to make the next three triangles will be:

10 + 5 = 15

15 + 6 = 21

21 + 7 = 28

15, 21, 28 dots.

3. Write down the sequence of natural numbers leaving remainder 1 on division by 3 and the sequence of natural numbers leaving remainder 2 on division by 3.

Ans. The numbers that leave 1 as remainder when divided by 3 are 1, 4, 7, 10, 13, …

(Sequence each with difference of 3 and starting from 1)

The numbers that leave 2 as remainder when divided by 3 are 2, 5, 8, 11, 14, …

(Sequence each with difference of 3 and starting from 2)

4. Write down the sequence of natural numbers ending in 1 or 6 and describe it in two other ways.

Ans.1, 6, 11, 16, 21, ……..

Numbers, each with difference of 5 and starting from 1.

Numbers, when divided by 5, leaves 1 as remainder.

5. A tank contains 1000 litres of water and it flows out at the rate of 5 litres per second. How much water is there in

the tank after each second? Write their numbers as a sequence.

Ans. Water in the tank initially = 1000 litre

Water in the tank after first second = 1000 – 5 = 995 litre

Water in the tank after next second

= 995 – 5 = 990 litre

Water in the tank after third second

= 990 – 5 = 985 litre

Sequence 1000, 995, 990, 985, ……

Page : 15

1. Write the algebraic expression for each of the sequences below:

i. Sequence of odd numbers

ii. Sequence of natural numbers which leave remainder 1 on division by 3.

iii. The sequence of natural numbers ending in 1.

iv. The sequence of natural numbers ending in 1 or 6.

Ans. i. Odd numbers are those which are not divisible by 2

i.e. 1, 3, 5… and so on

i.e. (2 - 1), (4 - 1), (6 - 1)…and so on

ie. X_n = 2n - 1, where n = 1, 2, 3… and so on

ii. Sequence of natural numbers which leave remainder 1 on division by 3 are 1, 4, 7, 10, 13, 16 .....

$x_1 = 1$

$x_2 = 1 + 3(1) = 4$

$x_3 = 1 + 3(2) = 7$

……

……..

$x_n = 1 + 3(n – l) = 3n – 2$

Algebraic expression $x_n = 3n – 2$ where, n = 1, 2, 3 ……..

iii. The sequence of natural numbers ending in 1 is 1, 11, 21, 31…….

That is $10 \times 1 – 9, 10 \times 2 – 9,$ …….

Algebraic expression xn = 10n – 9, where n = 1,2,3 ……

iv. The sequence of natural numbers ending in 1 or 6 is 1, 6, 11, 16, 21 ……

Observing the pattern, these numbers can be written as (5 - 4), (10 - 4), (15 - 4),…and so on

ie. $5 \times 1 – 4, 5 \times 2 – 4$,.....

Algebraic expression $x_n = 5n – 4$ where, n = 1, 2, 3 …….

2. For the sequence of regular polygons starting with an equilateral triangle, write the algebraic expressions for the sequence of the sums of inner angles, the sums of the outer angles, the measures of an inner angle, and the measure of an outer angle.

Ans. Let n be the number of sides

Sum of interior angles: 180°, 360°, 540°, 720°, ……

Algebraic expression : $x_n = 180n$

Sum of exterior angles 360°,360°,360°,..

Algebraic expression $x_n = 360°$

Sequence of one interior angle:

$\frac{180^o}{3}, \frac{360^o}{4}, \frac{540^o}{5}$, ...

Algebraic expression : $x_n = \frac{180^o n}{n+2}$

where, n = 1, 2, 3, ...

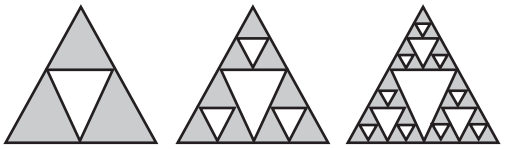
Sequence of one exterior angle:

$\frac{360^o}{3}, \frac{360^o}{4}, \frac{360^o}{5}$, ...

Algebraic expression : $x_n = \frac{360^o}{n+2}$,

where, n = 1, 2, 3, ...

3. Look at these pictures:



The first picture is got by removing the small triangle formed by joining the midpoints of an equilateral triangle. The second picture is got by removing such a middle triangle from each of the red triangles of the first picture. The third picture shows the same thing done on the second.

i. How many red triangles are there in each picture?

ii. Taking the area of the original uncut triangle as 1, compute the area of a small triangle in each picture.

iii. What is the total area of all the red triangles in each picture?

iv. Write the algebraic expressions for these three sequences obtained by continuing this process.

Ans. i. The red triangles in the first picture = 3

Red triangles in the second picture = 9

Red triangles in the third picture = 27

ii. Area of the original uncut triangle = 1

So area of small triangle in the first triangle = $\frac{1}{4}$

Area of red triangles in the second picture = $\frac{1}{16}$

Area of red triangles in the third picture = $\frac{1}{64}$

iii. Total area of red triangle in the first picture = $3 \times \frac{1}{4} = \frac{3}{4}$

Total area of red triangles in the second picture = $9 \times \frac{1}{16} = \frac{9}{16}$

Total area of red triangles in the third picture = $27 \times \frac{1}{64} = \frac{27}{64}$

iv. First sequence 3, 9, 27 ... $3^n = 3^1$, ..3^n

Algebraic expression = 3^n, where n = 1, 2, 3, ...

Second sequence $\frac{1}{4}, \frac{1}{16}, \ldots \frac{1}{4^n}$,

Algebraic expression = $\frac{1}{4^n} = \left(\frac{1}{4}\right)^n$

where, n = 1, 2, 3 ....

Third sequence : $3 \times \frac{1}{4}, 9 \times \frac{1}{16}, \ldots \left(\frac{3}{4}\right)^n$

Algebraic expression = $\left(\frac{3}{4}\right)^n$ where, n = 1, 2, 3 ....

Page : 18

1. Check whether each of the sequences given below is an arithmetic sequence. Give reasons. For the arithmetic sequences, write the common difference also.

i. Sequence of odd numbers.

ii. Sequence of even numbers.

iii. Sequence of fractions got as half the odd numbers.

iv. Sequence of powers of 2.

v. Sequence of reciprocals of natural numbers.

Ans. i. Sequence of odd numbers 1,3,5,7,...

Common difference = 2

$\therefore$ This is an arithmetic sequence.

ii.Sequence of even numbers 2,4, 6, 8,…

Common difference = 2

$\therefore$ This is an arithmetic sequence.

iii. Sequence of half of the odd numbers

$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$ = 0.5, 1.5, 2.5, ...

Common difference = $\frac{3}{2} - \frac{1}{2} = 1$

$\therefore$ This is an arithmetic sequence.

iv. Sequence of powers of 2 : $2^1, 2^2, 2^3, 2^4$,

2, 4, 8,16, ……

Difference of adjacent numbers

$= 4 - 2 \neq 8 - 4$

It does not have a common difference

$\therefore$ This is not an arithmetic sequence

v. Sequence of reciprocals of natural numbers

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

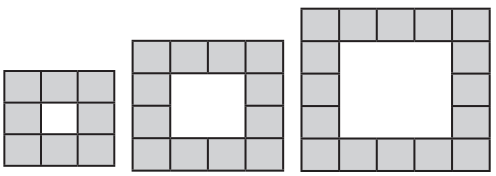
Difference of adjacent numbers,

$= \frac{1}{2} - 1 = \frac{-1}{2}$ and $\frac{1}{3} - \frac{1}{2} = \frac{-1}{6}$

It does not have a common difference

$\therefore$ This is not an arithmetic sequence

2. Look at these pictures:



If the pattern is continued, do the numbers of coloured squares form an arithmetic sequence? Give reasons.

Ans. Numbers of coloured squares in each picture 8, 12, 16, ……

Common difference = 12 – 8 = 4

$\therefore$ This is an arithmetic sequence

3. See the pictures below:

i. How many small squares are there in each rectangle?

ii. How many large squares?

iii. How many squares in all?

Continuing this pattern, is each such sequence of numbers, an arithmetic sequence?

Ans. i. No., of small squares in the first picture = 2

No.of small squares in the 2nd picture = 4

No. of small squares in the 3rd picture = 6

No. of small squares in the 4th picture = 8

ii. No. of big squares in first picture = 0

No. of big squares in second picture = 1

No. of big squares in third picture = 2

No. of big squares in fourth picture = 3

iii. Total squares in first picture = 2

Total squares in second picture = 5

Total squares in third picture = 8

Total squares in fourth picture = 11

Sequence of no. of small squares 2, 4, 6, 8

This is an arithmetic sequence with common difference = 2

Sequence of no. of big squares, 0, 1, 2, 3

This is an arithmetic sequence with common difference = 1

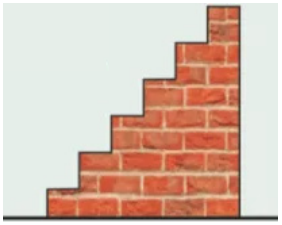
Sequence of Total no. of squares 2, 5, 8, 11

This is an arithmetic sequence with common difference = 3

4. In the staircase shown here the height of the first step is 10 centimetres and the height of each step after it is 17.5 centimetres.

i. How high from the ground would be some-one climbing up, after each step?

ii. Write these numbers as a sequence



Ans. i. Height from the ground after climbing up first step = 10 cm

second step = 10 + 17.5 = 27.5cm

third step = 27. 5 + 17.5 = 45 cm

fourth step = 45 + 17.5 = 62.5 cm

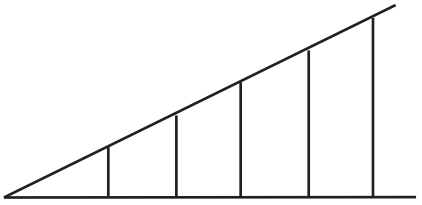
fifth step = 62.5 + 17.5 = 80 cm

sixth step = 80 + 17.5 = 97.5cm

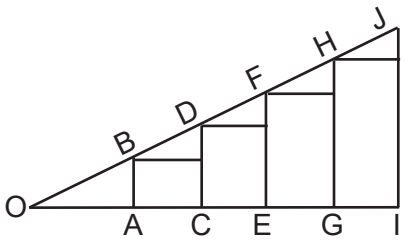
ii. Sequence of height

10, 27.5, 45, 62.5, 80, 97.5 ……

5. In this picture, the perpendiculars to the bottom line are equally spaced. Prove that, continuing like this, the lengths of perpendiculars form an arithmetic sequence.



Ans.



OA = AC = CE = EG = GI = x

In figure ΔOAB, ΔOCD are similar right-angled triangles. That is if one side and its included angles of a triangle are equal to the one side and included angles of another triangle, then the triangles are similar.

∴ sides are proportional.

$\frac{OA}{OC} = \frac{x}{2x} = \frac{AB}{CD} \Rightarrow CD = 2AB$

similarly, EF = 3AB.

So the sequence : AB, 2AB, 3AB,……..

That is AB is the common difference, so perpendicular lengths are in arithmetic sequence

6. The algebraic expression of a sequence is $x_n = n^3 - 6n^2 + 13n - 7$

Is it an arithmetic sequence?

Ans. Algebraic expression = $n^3 - 6n^2 + 13n - 7$

$n = 1 \Rightarrow 1 - 6 + 13 - 7 = 1$

$n = 2 \Rightarrow 8 - 24 + 26 - 7 = 3$

$n = 3 \Rightarrow 27 - 6 \times 9 + 39 - 7$

$= 27 - 54 + 32 = 5$

$n = 4 \Rightarrow 64 - 96 + 52 - 7 = 13$

1, 3, 5, 13,……. This is not an arithmetic sequence as it does not have a common difference.

Page : 21

1. In each of the arithmetic sequences below, some terms are missing and their positions are marked with ○. Find them.

i. 24, 42, ○, ○, ...

ii. ○, 24, 42, ○, ...

iii ○, ○, 24, 42, ...

iv. 24, ○, 42, ○, ...

v. ○, 24, ○, 42, ...

vi. 24, ○, ○, 42, ...

Ans. i. 24, 42, ○, ○, ...

Common difference = 42 – 24 = 18

So the missing numbers are :

42 + 18 = 60 and 60 + 18 = 78

Arithmetic sequence is 24, 42, 60, 78, …

ii. ○, 24, 42, ○, ...

Common difference = 42 – 24 = 18

So the missing numbers are :

24 – 18 = 6 and 42 + 18 = 60

Arithmetic sequence is 6, 24, 42, 60, …

iii. ○, ○, 24, 42, ...

Common difference = 18

So the missing numbers are :

24 – 18 = 6 and 6 – 18 = – 12

Arithmetic sequence is – 12, 6, 24, 42,..

iv. 24, ○, 42, ○, ...

Common difference =

(42 – 24)/2 = 18/2 = 9

So the missing numbers are :

24 + 9 = 33, 42 + 9 = 51

Arithmetic sequence is 24, 33, 42, 51 ….

v. ○, 24, ○, 42, ...

Common difference = (42 - 24)/2 = 18/2 = 9

So the missing numbers are :

24 – 9 = 15 and 24 + 9 = 33

Arithmetic sequence is 15, 24, 33, 42

vi. 24, ○, ○, 42, ...

24 + 3 common difference = 42

3 common difference = 42 - 24 = 18

common difference = 18/3 = 6

So the missing numbers are :

24 + 6 = 30 and 30 + 6 = 36

Arithmetic sequence is 24, 30, 36, 42, ...

2. The terms in two positions of some arithmetic sequences are given below.

Write the first five terms of each:

i. 3rd term 34 6th term 67

ii. 3rd term 43 6th term 76

iii. 3rd term 2 5th term 3

iv. 4th term 2 7th term 3

v. 2nd term 5 5th term 2

Ans. i. Third term = 34, Sixth term = 67

We get the 6th term from the 3 rd term, by adding the common difference (6 – 3 = 3) 3 times.

3 times of common difference

= 67 – 34 = 33

So Common difference = $\frac{33}{3}$ = 11

Second term = 34 – 11 = 23

First term = 23 – 11 = 12

∴ First five terms are 12, 23, 34, 45, 56

ii. Third term = 43, Sixth term = 76

Thrice the common difference = 76 – 43 = 33

Common difference = $\frac{33}{3}$ = 11

Second term = 43 – 11 = 32

First term = 32 – 11 = 21

First five terms are 21, 32, 43, 54, 65.

iii. Third term = 2, Fifth term = 3

Twice the common difference = 3 – 2 = 1

Common difference = $\frac{1}{2}$

Second term = $2 - \frac{1}{2} = 1\frac{1}{2}$

First term = $1\frac{1}{2} - \frac{1}{2} = 1$

First five terms are 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, 3

iv. Fourth term = 2, Seventh term = 3

Thrice the common difference =3 – 2 = 1

Common difference = $\frac{1}{3}$

Third term = $2 - \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$

Second term = $\frac{5}{3} - \frac{1}{3} = \frac{4}{3} = 1\frac{1}{3}$

First term = $\frac{4}{3} - \frac{1}{3} = 1$

First five terms are $1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}$

v. Second term = 5, Fifth term = 2

Thrice the common difference= 2– 5 = -3

Common difference = $\frac{-3}{3} = -1$

First term = 5 – (–1) = 6

First five terms are 6, 5, 4, 3, 2

3. The 5th term of an arithmetic sequence is 38 and the 9th term is 66. What is its 25th term?

Fifth term = 38, Ninth term = 66

Term difference = 66 – 38 = 28

Position difference = 9 – 5 = 4

Common difference = $\frac{28}{4} = 7$

25th term = Fifth term + 20 × Common difference = 38 + 20 × 7 = 38 + 140 = 178

4. Is 101 a term of the arithmetic sequence 13, 24, 35, …? What about 1001?

Ans. Common difference = 24 – 13 = 11

Difference between 101 and 13

= 101 – 13 = 88 = 8 × 11

That is we can get 101 by adding 8 times common difference with 13.

So, '101' is the 9th term of this arithmetic sequence.

Difference between 1001 and 13

= 1001 – 13 = 988

This is not a multiple of 11.

So '1001' is not a term of this arithmetic sequence.

5. How many three-digit numbers are there, which leave a remainder 3 on division by 7?

Ans. Numbers are : 101, 108, ……. 997

First term = 101, Common difference = 7

Last term = 997

Term difference = 997 – 101 = 896

= 128 × 7

Last term = First term + 128 × common difference

ie., 997 is the 129th term.

6. Fill up the empty cells of the given square such that the numbers in each row and column form arithmetic sequences:

1			4
7			28

What if we use some other numbers instead of 1, 4, 28 and 7?

Ans. 1. In the first row differences between 1st and 4th terrm = 3

Position difference = 4 – 1 = 3

Common difference = $\frac{3}{3} = 1$

Arithmetic sequence is 1, 2, 3,4 .....

2. In the first coloumn

Term difference = 7 – 1 = 6

Position difference = 4 – 1 = 3

Common difference = $\frac{6}{3} = 2$

Arithmetic sequence 1, 3, 5, 7…….

3. In the fourth row

Term difference = 28 – 7 = 21

Position difference = 4 – 1 = 3

Common difference = $\frac{21}{3}$ = 7

Arithmetic sequence is 7, 14, 21, 28 ...

4. In the second coloumn 2, –,– 14

Term difference = 14 – 2 = 12

Position difference = 4 – 1 = 3

Common difference = $\frac{14 - 2}{3}$ = 4

Arithmetic sequence is 2, 6, 10, 14....

5. In the third coloumn 3, –, –, 21

Term difference = 21 – 3 = 18

Position difference = 4 – 1 = 3

Common difference = $\frac{21 - 3}{3}$ = 6

Arithmetic sequence is 3, 9,15, 21.....

6. In the fourth coloumn 4, –, –, 28

Term difference = 28 – 4 = 24

Position difference = 4 – 1 = 3

Common difference = $\frac{28 - 4}{3}$ = 8

Arithmetic sequence is 4,12, 20, 28 ....

1	2	3	4
3	6	9	12
5	10	15	20
7	14	21	28

For other numbers instead of 1, 4, 28, 7

2			5
8			29

2	3	4	5
4	7	10	13
6	11	16	21
8	15	22	29

7. In the table below, some arithmetic sequences are given with two numbers against each. Check whether each belongs to the sequence or not.

Sequence	Numbers	Yes/No
11, 22, 33,...	123	
	132	
12, 23, 34,...	100	
	1000	
21, 32, 43,...	100	
	1000	
$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, ...$	3	
	4	
$\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, ...$	3	
	4	

Ans. i. Sequence 11, 22, 33, ……..

Difference = 22 – 11 = 11

Difference between 123 and 11 = 112

112 is not a multiple of 11, so 123 is not a term of this arithmetic series.

Difference between 132 and 11 = 121

121 is a multiple of 11, so 132 is a term of this arithmetic series.

ii. Sequence 12, 23, 34,....

Common difference = 23 – 12 = 11
Difference between 100 and 12 = 88

88 is a multiple of 11, so 100 is a term of this arithmetic series.

Difference between 1000 and 12 = 988

988 is not a multiple of 11, so 1000 is not a term of this arithmetic series.

iii. Sequence 21, 32, 43, ……

Common difference = 32 – 21 = 11

Difference between 100 and 21 = 79

79 is not a multiple of 11, so 1000 is not a term of this arithmetic series.

Difference between 1000 and 21 = 979

979 is a multiple of 11, so 1000 is a term of this arithmetic series.

iv. Sequence $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, ...$

Common difference = $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Difference between 3 and $\frac{1}{4}$

$= 3 - \frac{1}{4} = \frac{11}{4}$

$\frac{11}{4}$ is a multiple of $\frac{1}{4}$,

so 3 is a term of this arithmetic series.

Difference between 4 and $\frac{1}{4}$

$= 4 - \frac{1}{4} = \frac{15}{4}$

$\frac{15}{4}$ is a multiple of $\frac{1}{4}$,

so 4 is a term of this arithmetic series.

v. Sequence $\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, ...$

Common difference = $1\frac{1}{2} - \frac{3}{4}$

$= \frac{3}{2} - \frac{3}{4} = \frac{3}{4}$

Difference between 3 and $\frac{3}{4}$

$= 3 - \frac{3}{4} = \frac{9}{4}$

$\frac{9}{4}$ is a multiple of $\frac{3}{4}$,

so 3 is a term of this arithmetic series.

Difference between 4 and $\frac{3}{4}$

$= 4 - \frac{3}{4} = \frac{13}{4}$

$\frac{13}{4}$ is not a multiple of $\frac{3}{4}$,

so 4 is not a term of this arithmetic series.

Sequence	Numbers	Yes/No
11, 22, 33,...	123	No
	132	Yes
12, 23, 34,...	100	Yes
	1000	No
21, 32, 43,...	100	No
	1000	Yes
$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, ...$	3	Yes
	4	Yes
$\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, ...$	3	Yes
	4	No

Page : 24

1. The 8th term of an arithmetic sequence is 12 and its 12th term is 8. What is the algebraic expression for this sequence?

Ans. Difference between terms = 8 – 12 = -4

To get the 12th term from the 8th term, we must add the common difference (9 – 5 = 4) 4 times.

Common difference = – 4 / 4 = -1

Algebraic expression for arithmetic series, $x_n = an + b$

$12 = – 1 \times 8 + b \Rightarrow 12 = – 8 + b$

$b = 12 + 8 = 20$

Algebraic expression for arithmetic series = $x_n = –1 \times n + 20 = 20 – n$

2. The Bird problem in Class 8 (The lesson, Equations) can be slightly changed as follows.

One bird said:

"We and we again, together with half of us and half of that, and one more is a natural number"

Write all the possible number of birds starting from the least. For each of these, write the sum told by the bird also.

Find the algebraic expression for these two sequences.

Ans. If we take x as the number of birds

$x + x + \frac{x}{2} + \frac{x}{4} + 1$ is a natural number

$2x + \frac{3x}{4} + 1$ is a natural number

$\frac{11x}{4} + 1 = \frac{11x + 4}{4}$ is a natural number

$\therefore$ $11x + 4$ is a multiple of 4.

$\Rightarrow$ $11x$ is a multiple of 4.

$\Rightarrow x$ is a multiple of 4.

No. of birds 4, 8, 12, 16 ……

(a = 4, a + b = 4, b = 0)

Algebraic expression of the series

= an + b = 4n

No. of sums $\left(\frac{11x}{4} + 1, x = 4, 8, 12\ldots\right)$

= 12, 23, 34, 45,……

Algebraic expression for the series

= 12 + (n -1)11 = 11 n + 1

3. Prove that the arithmetic sequence with first term $\frac{1}{3}$ and common difference $\frac{1}{6}$ contains all natural numbers.

Ans. First term = 1/3, d = 1/6

$x_n = dn + (f - d) = \frac{1}{6} n + \frac{1}{6} = \frac{1}{6} (n+1)$

As n = 5, 11, 17,……., all natural numbers will occur in this arithmetic sequence

4. Prove that the arithmetic sequence with first term $\frac{1}{3}$ and common difference $\frac{2}{3}$ contains all odd numbers, but no even number.

Ans. $X_n = dn + (f - d) = \frac{2}{3} n + \left(\frac{1}{3} - \frac{2}{3}\right)$

$= \frac{2}{3} n - \frac{1}{3} = \frac{1}{3} (2n - 1)$

Value of (2n - 1) is always an odd number, Therefore there is no even number.

5. Prove that the squares of all the terms of the arithmetic sequence 4,7,10,…. belong to the sequence.

Ans. First term = 4, d = 3

$x_n = dn + (f - d) = 3n + (4 - 3) = 3n + 1$

Squares = $(3n + 1)^2 = 9n^2 + 6n + 1$

$= 3(3n^2 + 2n) + 1 = 3k + 1$

This is in the form 3k + 1, so squares of all terms are in this series.

6. Prove that the arithmetic sequence 5,8,11,……. contains no perfect squares.

Ans. First term, f = 5, d = 3

$x_n = dn + (f - d) = 3n + (5 - 3) = 3n + 2$

That is when we divide a perfect square with 3 we get remainder as 1 or 0. Here we can divide 3k + 2 by 3 to get 2 as remainder and it doesn't contain any perfect square.

7. Write the whole numbers in the arithmetic sequence $\frac{11}{8}, \frac{14}{8}, \frac{17}{8}$ …… Do they form an arithmetic sequence?

Ans. $\frac{11}{8}, \frac{14}{8}, \frac{17}{8}$,....

$a = \frac{11}{8}$, $d = \frac{14}{8} - \frac{11}{8} = \frac{17}{8} - \frac{14}{8} = \frac{3}{8}$

Algebraic expression = $x_n = dn + (f - d)$

$= \frac{3}{8} n + \left(\frac{11}{8} - \frac{3}{8}\right) = \frac{3}{8} n + 1$

If 'n' is a multiple of 8 then 4, 7, 10, … is a sequence having whole numbers

Page : 28

1. Write three arithmetic sequences with 30 as the sum of the first five terms.

Ans. Let the five terms be :

$x, x + 1, x + 2, x + 3, x + 4,$

ie. $x + x + 1 + x + 2 + x + 3 + x + 4 = 30$

$5x + 10 = 30$

$5x = 20$

$\Rightarrow x = 4$

The arithmetic sequence is 4, 5, 6, 7, 8

If the five terms are

$x, x + 2, x + 4, x + 6, x + 8$, then

$x + x + 2 + x + 4 + x + 6 + x + 8 = 30$

$5x + 20 = 30$

$5x = 30 - 20 = 10$

$\Rightarrow x = 2$

The arithmetic sequence is 2, 4, 6, 8, 10

If the five terms are

$x, x + 3, x + 6, x + 9, x + 12$, then

$x + x + 3 + x + 6 + x + 9 + x + 12 = 30$

$5x + 30 = 30$

$5x = 0$

$x = 0$

The arithmetic sequence is 0, 3, 6, 9,12

The arithmetic sequences whose sum of first five terms give 30 are:

4, 5, 6, 7, 8, ……..

2, 4, 6, 8, 10, ……..

0, 3, 6, 9, 12, …….

2. The first term of an arithmetic sequence is 1 and the sum of the first four terms is 100. Find the first four terms.

Ans. First term, $f = 1$

Let the first four terms be :

$f, f + d, f + 2d, f + 3d$

ie. $f + f + d + f + 2d + f + 3d = 100$

$4f + 6d = 100$

$\Rightarrow 2f + 3d = 50$

ie. $2 + 3d = 50$

$3d = 50 - 2 = 48$

$\Rightarrow d = 16$

First four terms are : 1, 17, 33, 49,

3. Prove that for any four consecutive terms of an arithmetic sequence, the sum of the two terms on the two ends and the sum of the two terms in the middle are the same.

Ans. First four terms of an arithmetic sequence be : $x, x + d, x + 2d, x + 3d$

Sum of the two terms on the two ends = $2x + 3d$

Sum of the two terms in the middle

$= (x + d) + (x + 2d) = 2x + 3d$

ie. they both are equal

4. Write four arithmetic sequences with 100 as the sum of the first four terms.

Ans. Let the first four terms be :

$x - 3d, x - d, x + d, x + 3d$, then

$x - 3d + x - d + x + d + x + 3d = 100$

$4x = 100$

$\Rightarrow x = 25$

i. If d = 1, the arithmetic sequence is 22, 24, 26, 28...

ii. If d = 2, the arithmetic sequence is 19, 23, 27, 31 ...

iii. If d = 3, the arithmetic sequence is 16, 22, 28, 34

iv. If d = 4, the arithmetic sequence is 13, 21, 29, 37

5. Write the first three terms of each of the arithmetic sequences described below:

i. First term 30; the sum of the first three terms is 300.

ii. First term 30; the sum of the first four terms is 300.

iii. First term 30; the sum of the first five terms is 300.

iv. First term 30; the sum of the first six terms is 300.

Ans. i. f = 30, Sum of first three terms = 300

In the arithmetic sequence sum of any three consecutive natural numbers is thrice the middle number.

$\therefore$ Second term = $\frac{300}{3} = 100$

$\therefore$ Common difference = 100 – 30 = 70

$\therefore$ Third term = 100 + 70 = 170

Sequence 30, 100, 170, …….

ii. f = 30, Sum of first four terms = 300

In four consecutive terms of an arithmetic sequence, the sums of the first and the last is equal to the sum of the second and the third.

First term + Fourth term = Second term + Third term = $\frac{300}{2} = 150$

$\therefore$ Fourth term = 150 – 30 = 120

30, ……, ……., 120

Term difference = 120 – 30 = 90

Position difference = 4 – 1 = 3

Common difference = $\frac{90}{3} = 30$

$\therefore$ Sequence = 30, 60, 90,……

iii. f = 30, Sum of first five terms = 300

Sum of the five consecutive terms of arithmetic sequence is five times of its middle term.

Third term = $\frac{300}{5} = 60$

Common difference = 15

$\therefore$ Sequence = 30, 45, 60, …

iv. f = 30, Sum of first six terms = 300

First term + Sixth term = Second term + Fifth term = Third term + Fourth term = $\frac{300}{3} = 100$

Sixth term = 100 – First term = 100 – 30 = 70

Term difference = 70 – 30 = 40

Position difference = 6 – 1 = 5

Common difference = $\frac{40}{5} = 8$

Sequence = 30, 38, 46,.....

6. The sum of the first five terms of an arithmetic sequence is 150 and the sum of the first ten terms is 550.

i. What is the third term of the sequence?

ii. What is the eighth term?

iii. What are the first three terms of the sequence?

Ans. i. Sum of first five terms = 150

Sum of the five consecutive terms of arithmetic sequence is five times of its middle term.

$\Rightarrow$ Third term = $\frac{150}{5} = 30$

ii. First term + Tenth term = Second term + Nineth term = Third term + Eighth term = Fourth term + Seventh term = Fifth term + Sixth term = $\frac{550}{5} = 110$

Third term + Eighth term = 110

Eighth term = 110 – Third term

= 110 – 30 = 80

iii. Third term = 30, Eighth term = 80

Term difference = 80 – 30 = 50

Position difference = 8 – 3 = 5

Common difference = $\frac{50}{5} = 10$

So sequence is 10, 20, 30,…….

7. The angles of a pentagon are in arithmetic sequence. Prove that its smallest angle is greater than 36°.

Ans. Let the smallest angle in the pentagon be x

$x + x + d + x + 2d + x + 3d + x + 4d = 540$

$5x + 10d = 540$,

$x + 2d = 108$

$x = 36°$, then $d = 36$,

angles are 36, 72, 108, 144, 180.

180° will not be an angle of a pentagon Since the sum of exterior and interior angle is 180°.

So the smallest angle in a pentagon will always be greater than 36°

Page : 35

1. Find the sum of the first 25 terms of each of the arithmetic sequences below.

   i. 11, 22, 33,…. ii. 12, 23, 34,…

   iii. 21, 32, 43,…. iv. 19, 28, 37,…

   vi. 1, 6, 11,….

Ans. i. 11, 22, 33, ...

Common difference, a = 22 - 11 = 11

First term = 11, a + b = 11

b = 11 - 11 = 0, n = 25

$$\text{Sum} = \frac{1}{2}\text{an}(n + 1) + nb$$

$$= \frac{1}{2} \times 11 \times 25 \times 26 + 25 \times 0 = 3575$$

ii. 12, 23, 34, ...

Common difference, a = 23 - 12 = 11

First term = 12, a + b = 12

b = 12 - 11 = 1, n = 25

$$\text{Sum} = \frac{1}{2}\text{an}(n + 1) + nb$$

$$= \frac{1}{2} \times 11 \times 25 \times 26 + 25 \times 1 = 3600$$

iii. 21, 32, 43, ...

Common difference, a = 32 - 21 = 11

First term = 21, a + b = 21

b = 21 - 11 = 10, n = 25

$$\text{Sum} = \frac{1}{2}\text{an}(n + 1) + nb$$

$$= \frac{1}{2} \times 11 \times 25 \times 26 + 25 \times 10 = 3825$$

iv. 19, 28, 37, ...

Common difference, a = 28 - 19 = 9

First term = 19, a + b = 19

b = 19 - 9 = 10, n = 25

$$\text{sum} = \frac{1}{2}\text{an}(n + 1) + nb$$

$$= \frac{1}{2} \times 9 \times 25 \times 26 + 25 \times 10 = 3175$$

vi. Common difference, a = 6 - 1 = 5

First term = 1, a + b = 1

b = 1 - 5 = -4, n = 25

$$\text{sum} = \frac{1}{2}\text{an}(n + 1) + nb$$

$$= \frac{1}{2} \times 5 \times 25 \times 26 + 25 \times -4 = 1525$$

2. What is the difference between the sum of the first 20 terms and the next 20 terms of the arithmetic sequence 6, 10, 14,…?

Ans. Sum of first n terms = $\frac{n}{2}(2a + n - 1)d$

Sum of first 40 terms

$$= \frac{40}{2}(2x6 + 40 - 1)4$$

$$= 20 \times (51) \times 4 = 4080$$

Sum of first 20 terms

$$= \frac{20}{2}(2x6 + 20 - 1)4$$

$$= 10 \times (31) \times 4 = 1240$$

Difference between first 40 and first 20 terms = 4080 - 1240 = 2840

$\therefore$ difference between the sum of the first 20 terms and the next 20 terms

= 2840 - 1240 = 1600

3. Calculate the difference between the sums of the first 20 terms of the arithmetic sequences 6, 10, 14,… and 15, 19, 23,……

Ans. 6, 10, 14,.....

Algebraic expression of the above arithmetic sequence = 4n + 2

Sum of first n terms = $[6 + (4n + 2)] \frac{n}{2}$

$= 4n + 2n^2$

Sum of first 20 terms

$= (4 \times 20) + (2 \times 20^2) = 880$

Given, 15, 19, 23, ……

Algebraic expression of the arithmetic sequence = 4n + 11

sum of first n terms = $[15 + (4n + 11)]\frac{n}{2}$

$= 13n + 2n^2$

sum of first 20 terms = $13 \times 20 + 2 \times 20^2$

$= 1060$

Differences = 1060 – 880 = 180

4. Find the sum of all three-digit numbers, which are multiples of 9.

Ans. First 3 digit number divisible by 9 = 108

Last 3 digit number divisible by 9 = 999

So the arithmetic sequence is :

108, 117,126, …… 999

Number of three digit numbers

$= \frac{999 - 108}{9} + 1 = 100$

sum = $\frac{100}{2}$ [108 + 999]

= 50 x 1107 = 55350

5. The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the nth term of each:

i. $n^2 + 2n$ ii. $2n^2 + n$

iii. $n^2 – 2n$ iv. $2n^2 – n$ v. $n^2 – n$

Ans. i. $n^2 + 2n$.

First term = $1^2 + 2 \times 1 = 3$

Sum of first two terms = $2^2 + 2 \times 2 = 8$

Second term = 8 – 3 = 5

The arithmetic sequence is 3, 5, 7...

n^{th} term = dn + (f – d) = 2n + (3 – 2)

= 2n + 1

ii. $2n^2 + n$

First term = 2 + 1 = 3

Sum of first two terms = $2 \times 2^2 + 2$

= 8 + 2 = 10

Second term = 10 – 3 = 7

The arithmetic sequence is 3, 7, 11…

f = 3, d = 7 - 3 = 4

n^{th} term, X_n = dn + (f - d)

= 4n + (3 - 4) = 4n - 1

iii. $n^2 – 2n$

First term = 1 – 2 = – 1

Sum of first two terms = $2^2 – 4 = 0$

Second term = 0- (-1) = 0 + 1 = 1

The arithmetic sequence is –1, 1, …

n^{th} term, X_n = dn + (f - d)

= 2n + (-1 -2) = 2n - 3

iv. $2n^2 – n$

First term = 2 – 1 = 1

Sum of first two terms = $2 \times 2^2 – 2$

= 8 – 2 = 6

Second term = 6 – 1 = 5

The arithmetic sequence is 1, 5, 9……

n^{th} term, $X_n = dn + (f - d)$

$= 4n + (1 - 4) = 4n - 3$

v. $n^2 - n$

First term = 1 – 1 = 0

Sum of first two terms = $2^2 - 2 = 2$

Second term = 2 – 0 = 2

The arithmetic sequence is 0, 2, 4....

n^{th} term = $X_n = dn + (f - d) = 2n - 2$

6. Calculate in head, the sums of the following arithmetic sequences.

i. $51 + 52 + 53 + ... + 70$

ii. $1\frac{1}{2} + 2\frac{1}{2} + ... + 12\frac{1}{2}$

iii. $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + ... + 12\frac{1}{2}$

Ans. i. $51 + 52 + 53 + ... + 70$

First term = 51, last term = 70

common difference = 1,

$n = \frac{70 - 51}{1} + 1 = 20$

$sum = \frac{20}{2}[51 + 70]$

$= 10 \times 121 = 1210$

ii. $1\frac{1}{2} + 2\frac{1}{2} + ... + 12\frac{1}{2}$

$= \frac{3}{2} + \frac{5}{2} + ... \frac{25}{2}$

$n = \frac{\frac{25}{2} - \frac{3}{2}}{\frac{5}{2} - \frac{3}{2}} + 1 = 12$

$sum = \frac{12}{2}\left[\frac{25}{2} + \frac{3}{2}\right] = 6 \times \frac{28}{2}$

$= 3 \times 28 = 84$

iii. $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + ... + 12\frac{1}{2}$

$= \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2}.... + \frac{25}{2}$

$= \frac{1}{2}[1 + 2 + 3 + ... + 25]$

$n = 25, x_1 = \frac{1}{2}, X_n = 12\frac{1}{2}$

$sum = \frac{25}{2}\left(\frac{1}{2} + 12\frac{1}{2}\right) = \frac{25 \times 13}{2} = 162.5$

7. Prove that the sum of any number of terms of the arithmetic sequence 16, 24, 32,….. starting from the first, added to 9 gives a perfect square

Ans. 16, 24, 32,…….

Algebraic expression of the arithmetic sequence = 8n + 8

sum of n terms = $[16 + 8n + 8] \times \frac{n}{2}$

$= (24 + 8n) \times \frac{n}{2} = 12n + 4n^2$

Sum after adding 9 with n^{th} term

$= 4n^2 + 12n + 9 = (2n + 3)^2$

Hence it is a perfect square

8. 1
2 3
4 5 6
7 8 9 10
.....
.....

i. Write the next two lines of the pattern above.

ii. Write the first and the last numbers of the 10th line.

iii. Find the sum of all the numbers in the first ten lines.

Ans. i. First row contains one element, second row contains two elements, so fifth row will contain five elements and sixth row will contain six elements.

Consider the first row, 1, 2, 4, 7……..

= 1, 1 + 1, 2 + 2, 4 + 3, ……

= 1, 1 + 1, 2 + 2, 4 + 3, 7 + 4, 11 + 5 ……..

So 1, 2, 4, 7, 11, 16 ……generally it written as :

1 + 1 (1 + 2 + 3 + …….)

First term in the fifth row = 1 + 1(1 + 2 + 3 + 4) = 1 + 10 = 11

First term in the sixth row= 1 + 1(1 + 2 + 3 + 4 + 5) = 1 + 15 = 16

Common differences in each row = 1

Fifth row 11, 12, 13, 14, 15.

Sixth row 16, 17, 18, 19, 20, 21.

1
2 3
4 5 6
7 8 9 10
11 12 13 14 15
16 17 18 19 20 21
.....
.....

ii. First term in the tenth row = 1 + 1(1 + 2 + 3+ 4 + 5 + 6 + 7 + 8 + 9)

$= 1 + \frac{1 \times 9 \times 10}{2} = 46$

Last term in the tenth row = 46 + 1 × 9 = 46 + 9 = 55

iii. Numbers in the first ten lines 1, 2, 3, 4, 5, …….. 55

Sum of all the numbers in the first ten lines = $\frac{n(n+1)}{2} = \frac{55 \times 56}{2} = 1540$

9. 4
7 10
13 16 19
22 25 28 31
.....
.....

Write the next two lines of the pattern above. Calculate the first and last terms of the 20th line.

Ans. 4, 7, 13, 22, 34, 49,…….

(4, 4 + 3, 7 + 6, 13 + 9, 22 + 12, 34 + 15),

Generally it written as 4 + 3(1 + 2 + 3 + ……..)

First term in the fifth row = 4 + 3(1 +2 + 3 + 4) = 4 + 30 = 34

First term in the sixth row =

4 + 3(1 + 2+ 3 + 4 + 5) = 4 + 45 = 49

Common differences in each row = 3

Fifth row 34, 37, 40, 43, 46,..

Sixth row 49, 52, 55, 58, 61, 64…

4
7 10
13 16 19
22 25 28 31
34 37 40 43 46
49 52 55 58 61 64
.....

First term in the 20th row

= 4 + 3(1 + 2 + 3 + 4 +……. + 19)

$= 4 + \frac{3 \times 19 \times 20}{2} = 574$

Last term in the 20th row = 574 + 19 x 3

= 574 + 57 = 631

ADDITIONAL QUESTIONS & ANSWERS

1. —, 18, —, 28 are four consecutive terms of an arithmetic sequence. Fill in the blanks.

Ans. 18 + 2d = 28

2d = 28 – 18 = 10

$\Rightarrow$ d = 5

Sequence 13, 18, 23, 28.

2. 98 is a term of the arithmetic sequence having common difference 7. Is 2016 a term of this sequence. Why?

Ans. If (2016 – 98) is a multiple of common difference 7, then 2016 is a term of the arithmetic sequence.

2016 – 98 = 1918

1918 is a multiple of 7.

(Quotient = 274, Remainder = 0)

$\therefore$ 2016 is a term of this sequence.

3. For the arithmetic sequence 6, 12, 18,…

a. What is the common difference?

b. Find the 10th term?

Ans. a. Common difference = 12 - 6 = 6

b. 10^{th} term = f + 9d = 6 + (9 × 6)

= 6 + 54 = 60

4. The algebraic form of an arithmetic sequence is 3 + 2n.

a. What is the first term of the sequence?

b. What will be the remainder if the terms of the sequences are divided by 2?

Ans. a. First term = 3 + 2 x 1 = 5

b. d = 2 (coefficient of n be the common difference)

The remainder when divided by 2 = 1

5. The n^{th} term of an arithmetic sequence is $a_n = 5 – 6n$. Find its sum of n terms?

Ans. n^{th} term = 5 - 6 n

1^{st} term = 5 - 6 x 1 = 5 - 6 = -1

$$S = \frac{n}{2}[f + \ell] = \frac{n}{2}[-1 + 5 - 6n]$$

$$= \frac{n}{2}[4 - 6n] = n(2 - 3n) = 2n - 3n^2$$

6. Consider the multiples of 7 in between 100 and 500.

a. What are the first and last numbers?

b. How many terms are there in the sequence?

Ans. a. First term = 100 – 2 + 7 = 105

Last term = 500 – 3 = 497

b. n^{th} term, $x_n = dn + (f-d)$

497 = 7n + (105 - 7)

$$n = \frac{(497 - 98)}{7} = 57$$

So, number of terms = 57

7. For an arithmetic sequence 22, 26, 30, ..

a. What is the common difference?

b. Will 50 be a term of this sequence? Why?

c. Can the difference between any two terms of this sequence be 50? Justify your answer?

Ans. a. Common difference, d = 26 – 22 = 4

b. $\frac{50 - 22}{4} = 7$

So, 50 is a term of this sequence.

c. 50 is not a multiple of 4. So, 50 is cannot be a difference of two terms.

8. Find the smallest 3 digit number which is the multiple of 6. Find the sum of all the three-digit numbers which are the multiple of six.

Ans. Smallest 3 digit number which is the multiple of 6 = 102.

Highest number = 996

Common difference = 6

Arithmetic series : 102, 108, ….. 996

Number of three digit numbers

$$= \frac{996 - 102}{6} + 1 = 150$$

Sum = $\frac{150}{2}$ (102 + 996) = 82350

9. $5^2 \times 5^4 \times 5^6 \times ... \times 5^{2n} = (0.008)^{-30}$. Find n.

Ans. $5^2 \times 5^4 \times 5^6 \times ... \times 5^{2n} = (0.008)^{-30}$ ___(1)

$5^2 \times 5^4 \times 5^6 \times ... \times 5^{2n} = 5^{2 + 4 + 6 + ...2n}$

$= 5^{2 (1 + 2 + 3 + ...n)}$

$= 5^{2\left(\frac{n(n+1)}{2}\right)} = 5^{n(n+1)}$

Now, $(0.008)^{-30} = \left(\frac{8}{1000}\right)^{-30} = (5^{-3})^{-30} = 5^{90}$

(1) $\Rightarrow 5^{n(n + 1)} = 5^{90}$

$\Rightarrow$ n (n + 1) = 90

$n^2 + n - 90 = 0$

$\Rightarrow$ n = 9

10. Find the sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$.

Ans. $X_n = 3 + 2n$

Now, put n = 1, 2, 3, ...

$x_1 = 3 + 2 \times 1 = 5$

$x_2 = 3 + 2 \times 2 = 7$

$x_3 = 3 + 2 \times 3 = 9$

Thus, the terms of the AP are 5, 7, 9...

Here, f = 5 and d = 2

$S_{24} = \frac{24}{2}$ [2 x 5 + (24 - 1) x 2]

= 12 [10 + 46] = 672

11. a. Write the arithmetic sequence with first term 2 and common difference 3.

b. Check whether 100 is a term in this sequence.

c. Check whether the difference of any two terms of this sequence will be 2015.

d. Find the position of the term 125 in this sequence.

Ans. a. f = 2, d = 3

Sequence is : 2, 5, 8, 11, …….

b. If (100 – 2) is not a multiple of common difference 3, then 100 is not a term of the arithmetic sequence.

c. 2015 is not a multiple of common difference 3, so 2015 will not be the difference of any two terms of this sequence.

d. $x_n = 3n - 1$

$\Rightarrow 125 = 3n - 1$

ie. $3n = 126 \Rightarrow n = 42$

12. When 60 added to the first term of an A.P, we get its 11th term. Which number should be added to its first term to get the 19th term? Can 75 be the difference between any of the two terms of this sequence?

Ans. Differences between first term and 11th term is = 60

10 times of common difference = 60

$\therefore$ Common difference = 60/10 = 6

Differences between first term and 19th term is

= 18 x common difference = 18 x 6 = 108

When 108 is added to the first term, we get 19th term. The difference between two terms in an A.P is the multiple of common difference.

75 is not the multiple of common difference 6. So 75 cannot be the difference between two terms in the series.

13. i. What is the sum of first 20 natural numbers?

ii. The algebraic form of an arithmetic sequence is 6n + 5. Find the sum of first 20 terms of this sequence?

Ans. i. Sum of first n natural numbers

$= \frac{n(n + 1)}{2}$

$\therefore$ sum of first 20 natural nmbers

$= \frac{20 \times (20 + 1)}{2} = \frac{(20 \times 21)}{2}$

$= \frac{420}{2} = 210$

ii. The algebraic form = 6n + 5

common difference = coefficient of n = 6

First term 'x_1' = 6 x 1 + 5 = 11

Sum of first 20 terms = $\frac{1}{2}$ an(n + 1) + nb

$= \frac{1}{2} \times 6 \times 20 \times 21 + 20 \times 5$

$= 3 \times 20 \times 21 + 100$

$= 1260 + 100 = 1360$

14. 23rd term of an arithmetic sequence is 32. 35th term is 104. Then

a. What is the common difference?

b. Which is the middle term of first 35 terms of this sequence?

c. Find the sum of first 35 terms of this sequence.

Ans. a. 23rd term = 32, 35th term = 104

$x_{23} + 12d = x_{35}$

12d = 104 - 32 = 72

$\Rightarrow$ d = 6

b. a + 22 d = 32

a = 32 - 22 x 6 = - 100

Middle term of first 35 terms = 18 th term

= -100 + 17 x 6 = $\sqrt{4}$ = 2

OR

18 th term = $X_{19} = X_{23} - 5d$

$= 32 - 5 \times 6 = 32 - 30 = 2$

c. Sum of 35 terms = 18th term × 35 = 70

15. The difference between the 15th term and the 5th term of an A.P is 40. Which number is to be added to its 12th term.

Ans. The difference between the 15th term and the 5th term = 40

i.e., ten times of common difference = 40

Common difference = $\frac{40}{10} = 4$

When we add 8 x common difference we get 20th term.

The difference between the 12th term and the 20th term

= (20 – 12) × common difference

= 8 x 4 = 32

The difference between the first term and the 21th term

= (21 – 1) × Common difference

= 20 × 4 = 80

16. Consider the arithmetic sequence 171, 167, 163,……..

i) Is '0' is a term of this sequence? Why?

ii) How many positive terms are in this sequence?

Ans. Arithmetic series : 171, 167, 163 ……

i. Here common difference

d = $x_2 - x_1$ = 167 - 171 = -4

To check whether 0 is a term of this sequence, we have to find whether
0 - 171 = -171 is a multiple of common difference -4.

Since -4 is not a multiple of common difference, 0 will not be a term of the sequence

ii. 3 is the last positive term of the sequence.

$\therefore$ sum of positive terms of the sequence

$= \frac{3 - 171}{-4} + 1 = (\frac{-168}{-4}) + 1 = 42 + 1 = 43$

17.
1
2 3 4
5 6 7 8 9
.....
.....

a. How many numbers are there in the 30th row of this number pyramid?

b. Which is the last number in the 30th row?

c. Which is the first number in the 30th row?

d. What is the sum of all terms in the first 30 rows?

Ans. a. Total numbers in each sequence can be written as 1, 3, 5, …. $x_n = 2n - 1$

Numbers in the 30th row = 2 x 30 – 1 = 59

b. Last number in the first row = $1^2 = 1$

Last number in the second row = $2^2 = 4$

Last number in the third row = $3^2 = 9$

Last number in the 30th row = 30^2 = 900.

c. Number of terms in the 30th row = 59

Last number in the 30th row = 900

First number in the 30th row + 58d =

Last term in the 30th row

First number in the 30th row + 58 x 1 = 900

First number in the 30th row

= 900 – 58 = 842

d. The sum of all terms in the first 30 rows

$= \frac{900 \times 901}{2} = 450 \times 901 = 405540$

18. Write the sequence obtained by adding two adjacent consecutive terms in counting numbers starting 1. Write the algebraic expression of this sequence.

Ans. Counting numbers : 1, 2, 3, 4, 5. ....

Sequence obtained by adding two adjacent consecutive terms : 1 + 2, 2 + 3, 3 + 4, 4 + 5, ...

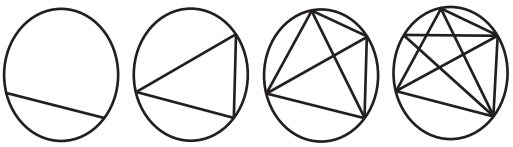
Algebraic expression of above sequence : $n + (n + 1) = 2n + 1$

19. Consider circles points on its circumference and chords shown in the figure. Mark two points on the circle and draw a chord. Mark one more points and draw three chords. Continue this process by adding one more point each time.

(a) Write the number of chords in each figure as a sequence.

(b) Write the algebraic expression of this sequence.

(c) Find the number of chords on the 10th figure.



Ans. (a) No. of chords in figure .1 = 1

No. of chords in figure. 2 = 1 + 2 = 3

No. of chords in figure. 3 = 1 + 2 + 3 = 6

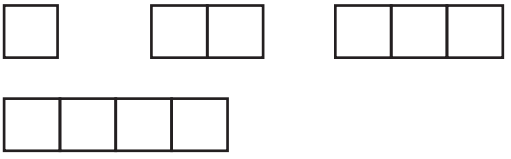
Sequence of number of chords = 1, 3, 6, 10, ...

(b) No. of chords in figure n, = 1 + 2 + 3 + ... + $n = \frac{n(n+1)}{2}$

(c) No. of chords in the 10th figure

$= \frac{10 \times 11}{2} = 55$

20. A pattern is formed using sticks of equal length is shown below:



(a) Write the sequence of number of

sticks used in each figure.

(b) Write the sequence of number of sequence rectangles in each figure.

(c) Write the algebraic expression in the above two sequences.

(d) Find the number of sticks and squares in the 10th figure.

Ans. (a) No. of sticks in the figure 1 = 1 + 3 = 4

No. of sticks in the figure 2 = 1 + 3 + 3 = 1 + 2 x 3 = 7

No. of sticks in the figure 3 = 1 + 3 x 3 = 10

No. of sticks in the figure 4 = 1 + 4 x 3 = 13

Sequence of number of sticks = 4, 7, 10, 13, ...

(b) No. of squares and rectangles in the figure . 1 = 1

No. of squares and rectangles in the figure . 2 = 2 + 1 = 3

No. of squares and rectangles in the figure . 3 = 3 + 2 + 1 = 6

No. of squares and rectangles in the figure, 4 = 4 + 3 + 2 + 1 = 10

Sequence of squares and rectangles = 1, 3, 6, 10, ...

(c) No. of sticks in the n^{th} figure = 1 + n x 3 = 3n + 1

No. of squares and rectangles in the n^{th} figure = 1 + 2 + 3 ... + n = $\frac{n\,(n+1)}{2}$

(d) No. of sticks in the 10th figure = 3 x 10 + 1 = 31

No. of squares and rectangles in the 10th figure = $\frac{10 \text{ x } 11}{2}$ = 55

21. Consider an arithmetic sequence with common difference 6 and 7th term 52. Find the 15th term of the arithmetic sequence. Is it possible to get a difference of 100 between any two terms of this sequence?

Ans. 15th term can be obtained by adding 8 times the common difference to the 7th term

i.e., $x_{15} = x_7 + 8d = 52 + 8 \text{ x } 6 = 100$

The difference between any two terms of an Arithmetic sequence will be a multiple of common difference.

100 can't be the difference between any two terms of this sequence, since it is not a multiple of 6.

22. Consider an arithmetic sequence whose 7th term is 34 and 15th term is 66.

(a) Find the common difference.

(b) Find the 20th term.

Ans.(a) 15th term can be obtained by adding 7th term and 8 times the common difference.

$x_{15} = x_7 + 8d$

$66 = 34 + 8d$

$8d = 66 - 34 = 32$

$d = \frac{32}{8} = 4$

(b) 20th term can be obtained by adding 15th term and 5 times the common difference.

$x_{20} = x_{15} + 5d = 66 + 5 \text{ x } 4 = 86$

23. Consider an arithmetic sequence $\frac{17}{7}$, $\frac{20}{7}$, $\frac{23}{7}$, ...

(a) Write the algebraic expression of the sequence.

(b) Write the sequence of counting numbers in the above given sequence. Is the newly obtained sequence an arithmetic sequence.

Ans.(a) Common Difference = $\frac{20}{7} - \frac{17}{7} = \frac{3}{7}$

Algebraic expression of sequence

$= \frac{17}{7} + (n - 1)\frac{3}{7}$

$x_n = \frac{3}{7}n + \frac{17}{7} - \frac{3}{7} = \frac{3}{7}n + 2$

(b) $x_n = \frac{3}{7}n + 2$

$x_7 = \frac{3}{7} \times 7 + 2 = 5$

$x_{14} = \frac{3}{7} \times 14 + 2 = 8$

$x_{21} = \frac{3}{7} \times 21 + 2 = 11$

$x_n = \frac{3}{7} \times n + 2$

If we give multiples of 7 for n in the expression $\frac{3}{7}n + 2$, we get counting numbers.

i.e., $x_n = \frac{3}{7} \times 7m + 2 = 3m + 2$

Thus 5, 3, 11.... is an arithmetic sequence with common difference 3

24. Considering an arithmetic sequence $\frac{17}{7}, \frac{31}{7}, \frac{45}{7}, ...$

(a) Write the algebraic expression of the sequence.

(b) Is there any counting number in this sequence? Justify your answer?

Ans.(a) Common difference= $\frac{31}{7} - \frac{17}{7} = \frac{14}{7} = 2$

Algebraic expression of the sequence

$= \frac{17}{7} + (n - 1)\ 2 = 2n + \frac{17}{7} - 2 = 2n + \frac{3}{7}$

For any counting number 'n', $2n$ will be counting number. But when $\frac{3}{7}$ is added to $2n$ it becomes a fraction. So this sequence will not have a counting number.

25. Find the 20th term of an arithmetic sequence if its 6th term is 14 and 14th term is 6.

Ans. $x_6 = 14; x_{14} = 6$

$x_{14} - x_6 = (14 - 6)\ d$

$6 - 14 = 8d$

$\Rightarrow d = -1$

$x_{20} = x_{14} + 6d = 6 + 6 \times -1 = 0$

26. Consider an arithmetic sequence whose m^{th} terms is 'n' and n^{th} term is 'm'.

(a) Find the common difference of the sequence

(b) Prove that $(m + n + p)^{th}$ term of the sequence is '$-p$'

Ans. (a) $x_m = n; x_n = m$

$x_m - x_n = (m - n)\ d$

$n - m = (m - n)\ d$

$d = \frac{n - m}{m - n} = \frac{-(m - n)}{m - n} = -1$

(b) $x_{m+n+p} = x_m + (m + n + p - m)\ d$

$= n + (n + p)\ x - 1$

$= n - n - p = -p$

27. Find the 13th term of an arithmetic sequence if 5 times the 5th term is equal to 8 times the 8th term.

Ans. 8th term = $x_5 + 3d$

$5x_5 = 8\ (x_5 + 3d)$

$5x_5 = 8x_5 + 24d$

$-3x_5 = 24d$

$x_5 = -8d$

$x_{13} = x_5 + 8d = -8d + 8d = 0$

28. Consider the arithmetic sequence 10, 16, 22, ... Can you find out any terms as

the sum or difference of any two terms of this sequence.

Ans. Algebraic expression of the sequence $x_n = 6n + 4$

Each term when divided by 6, remainder is 4.

n^{th} term = $6n + 4$

m^{th} term = $x_m = 6m + 4$

$x_n + x_m = 6n + 4 + 6m + 4$

$= 6(n + m) + 8 = 6(n + m) + 6 + 2$

$\Rightarrow x_n + x_m$ when divided by 6 will give remainder '2'. Therefore $x_n + x_m$ can't be a term in the given sequence.

Similarly $x_n - x_m = (6n + 4) - (6m + 4)$

$= 6n - 6m = 6(n - m)$

$x_n - x_m$ when divided by 6 will give remainder '0'. Therefore $x_n - x_m$ can't be a term in the given sequence.

29. Prove that the square of any term of the arithmetic sequence 7, 11, 10,..... will not be a term of the sequence.

Ans.• Algebraic expression of the sequence $x_n = 4n + 3$.

$\Rightarrow$ Each term when divided by 4 will leave the remainder 3.

$x_n^2 = (4n + 3)^2 = 16n^2 + 24n + 9$

$= 16n^2 + 24n + 8 + 1$

x_n^2 when divided by '4' will leave remainder '1'.

$\Rightarrow x_n^2$ will not be a term of the given sequence.

30. Find the 110^{th} term in the arithmetic sequence 5, 12, 19, ...

Ans. $a_n = a + (n - 1) \times d$

$a_{110} = 5 + 109 \times 7$

$= 5 + 763 = 768$

31. Consider two arithmetic sequences given below: 11, 19, 27, ... and 50, 55, 60, ...

Is there a common number to these sequences at same term position? If yes, find the term positions. Find the term?

Ans. For the sequence 11, 19, 27, ...;

$x_n = 8n + 3$

For the sequence 50, 55, 60, ...;

$x_n = 5n + 50 - 5 = 5n + 45$

If n^{th} terms are equal for both sequences

$8n + 3 = 5n + 45$

$8n - 5n = 45 - 3 = 42$

$3n = 42$

$n = \frac{42}{3} = 14$

The 14^{th} term of both sequences are equal.

14^{th} term = $8 \times 14 + 3 = 112 + 3 = 115$.

32. Consider the arithmetic sequence -74, -68, -62, ... How many negative numbers are there in this sequence? Find the first positive number in this sequence?

Ans. Common difference = 6

Algebraic expression of the sequence,

$x_n = 6n - 80$

If zero is a term in the sequence, then

$6n - 80 = 0$

$6n = 80$

$n = \frac{80}{6} = 13\frac{2}{6}$

Here n is not a natural number. This shows that zero is not a term in the sequence. Hence upto zero there are 13 terms. So there are 13 negative numbers in this sequence.

The first positive number will be 14th term
$= x_{14} = 6 \times 14 - 80 = 84 - 80 = 4$

33. $2x + 1$, $4x - 1$, $5x + 1$, ... are in an arithmetic sequence.

(a) Find x?

(b) Write the algebraic expression of the sequence.

(c) Find the position of 195 in this sequence.

Ans. (a) If a, b, c are three consecutive terms of an arithmetic sequence, then

$2b = a + c$

$8x - 2 = 7x + 2$

$x = 4$

$\therefore$ the sequence is 9, 15, 21, ...

(b) The algebraic expression of this sequence: $x_n = 6n + 3$

(c) $6n + 3 = 195$

$6n = 195 - 3 = 192$

$n = \frac{192}{6} = 32$

$\therefore$ 32nd term of the sequence is 195

34. (a) Find the sum of first 25 counting numbers

(b) Find the sum of first 25 even numbers

(c) Find the sum of first 25 odd numbers

Ans. (a) $1 + 2 + 3 + ... + 25 = \frac{25 \times 26}{2}$

$= 25 \times 13 = 325$

(b) $2 + 4 + 6 + ... + 50 = 2 (1 + 2 + 3 + ... + 25) = 2 \times \frac{25 \times 26}{2} = 650$

(c) $1 + 3 + 5.... + 49$

$= (2 - 1) + (4 - 1) + (6 - 1) + ... + (50 - 1)$

$= 2 + 4 + 6 + ..+ 50 - (1 + 1 +...+ 1)$

$= 2 (1 + 2 + 3 + ... + 25) - (1 \times 25)$

$= 650 - 25 = 625$

35. Prove that the sum of first n odd numbers is n^2.

Ans. • $1 + 2 + 3 + ... + n = \frac{n (n + 1)}{2}$

• $1 + 3 + 5 + .. + 2n - 1 = (2 - 1) + (4 - 1) + (6 - 1) + (2n - 1)$

$= (2 + 4 + 6 .... + 2n) - (1 + 1 + 1 +...+ 1)$ (n times)

$= 2 (1 + 2 + 3 ..... + n) - (1 \times n)$

$= 2 \frac{n (n + 1)}{2} - n = n^2 + n - n = n^2$

36. Find the sum of first 25 terms of the arithmetic sequence 5, 8, 11, ...

Ans. $S_n = \frac{n}{2} [2a + (n - 1)d]$

Here n = 25, a = 5, d = 3

$\therefore S_{25} = \frac{25}{2} [10 + 24 \times 3]$

$= \frac{25}{2} \times 82 = 1025$

37. Consider an arithmetic sequence whose 6th term is 40 and 9th term is 58.

(a) Find the 25th term of the sequence

(b) Find the sum of first 25 terms of the sequence

(c) Find the sum of first n terms of the sequence

Ans. (a) $x_9 = x_6 + 3d$

$58 = 40 + 3d$

$3d = 58 - 40 = 18$

$\Rightarrow d = 6$

$x_1 = x_6 - 5d = 40 - 5 \times 6 = 10$

$x_{25} = x_6 + 19d = 40 + 19 \times 6 = 154$

(b) Sum of first 25 terms $= \frac{25}{2} (x_1 + x_{25})$

$= \frac{25}{2}(10 + 154)$

$= \frac{25}{2} \times 164 = 2050$

(c) nth term $x_n = 6n + 4$

Sum of first 'n' terms = $\frac{n}{2}(x_1 + x_n)$

$= \frac{n}{2}(10 + 6n + 4)$

$= n(3n + 7) = 3n^2 + 7n$

38. Let the algebraic expression of an arithmetic sequence be $6n + 3$.

(a) Find the sum of first 20 terms of the sequence

(b) Write the algebraic expression of the sum.

Ans. Given : $x_n = 6n + 3$

(a) Sum of n terms = $\frac{6n(n+1)}{2} + 3n$

Sum of 20 terms

$= \frac{6 \times 20 \times 21}{2} + 3 \times 20 = 1260 + 60 = 1320$

(b) Sum of n terms = $\frac{6n(n+1)}{2} + 3n$

$= 3n^2 + 3n + 3n = 3n^2 + 6n$

39. (a) Find the sum of first 20 terms of counting numbers.

(b) Consider an arithmetic sequence whose common difference is '7' and sum of first 20 terms is '1530'. Write the algebraic expression of the sequence.

Ans. (a) $1 + 2 + 3 + ... + 20 = \frac{n \times (n+1)}{2}$

$= \frac{20 \times 21}{2} = 210$

(b) The n^{th} term of an arithmetic sequence with common difference 7 is $x_n = 7n + b$

Sum of first '20' terms = $\frac{7n(n+1)}{2} + b\,n$

$= 7 \times 10 \times 21 + b \times 20 = 1470 + b \times 20$

$\Rightarrow 1470 + b \times 20 = 1530$

$20b = 60 \quad \Rightarrow b = 3$

So the algebraic expression of the sequence is $7n + 3$

40. Consider an arithmetic sequence whose sum of first 10 terms is 250 and sum of first 16 terms is 592

(a) Write the algebraic expression of the sequence

(b) Write the algebraic expression of the sum of the sequence

Ans. (a) Assume that the algebraic expression of the sequence $x_n = an + b$

Sum of first 10 terms

$= a \times \frac{10 \times 11}{2} + b \times 10 = 10\left(\frac{11}{2}a + b\right)$

$\Rightarrow \frac{11}{2}a + b = 25$ _____ (1)

Sum of first 10 terms

$= a \times \frac{16 \times 17}{2} + b \times 16 = 16\left(\frac{17}{2}a + b\right)$

$\Rightarrow \frac{17}{2}a + b = 37$ _____ (2)

(2) - (1) $\Rightarrow 3a = 12$

$\Rightarrow a = 4$

(1) $\Rightarrow \frac{11}{2} \times 4 + b = 25$

$b = 3$

So the algebraic expression,

$x_n = an + b = 4n + 3$

(b) Assume that the algebraic expression for the sum of first n terms,

$S_n = \frac{an(n+1)}{2} + bn$

$= \frac{4n(n+1)}{2} + 3n$

$= 2n(n+1) + 3n$

$= 2n^2 + 2n + 3n = 2n^2 + 5n$

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